

Nova višinska referenčna ploskev na območju severne Alžirije, določena na podlagi modela geoida EGM2008/RTM

EGM2008/RTM based geoid model as support to a new vertical reference surface over northern part of Algeria

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IZVLEČEK

V članku je opisana določitev nove vertikalne referenčne ploskve v severnem delu Alžirije na osnovi globalnega modela geoida EGM2008/RTM kot podpora za potresne in geodinamične aplikacije. Za določitev pogreška neupoštevanja visokofrekvenčnega dela spektra težnostnega polja smo uporabili tehniko RTM (Residual Terrain Model) in 2D model bočnih kontrastov gostote zemeljskih mas, ki je bil pridobljen s pomočjo geoloških kart. Iz primerjave samo EGM2008 geoidnih višin z višinami, določenimi z metodo GPS/nivelman, smo dobili standardne odklone 8,5 cm pred oz. 5,5 cm po prilagoditvi modela z linearno korekcijsko ploskvijo. Uporaba ocene neupoštevanja visokofrekvenčnega dela spektra na osnovi tehnike RTM za globalni model EGM2008, je zmanjšala standardni odklon na 2,0 cm, kar je prispevalo izboljšavi modeliranja geoida za skoraj 63,5 %. Poleg tega, opisani geoidni model, ki temelji na tehniki EGM2008/RTM je v primerjavi z drugimi novejšimi lokalnimi in afriškimi gravimetričnimi geoidnimi modeli bolj zanesljiv. Nato smo izvedli raziskavo korelacije med topografskimi podatki, 2D modelom lateralnih variacij gostote, značilnimi parametri težnostnega polja in seizmološkimi podatki. Rezultati kažejo na pomembno stopnjo korelacije med geoidnimi višinami in magnitudo potresa, ki je približno 40 % za potrese z magnitudo 5 in več po Richterjevi lestvici.

KLJUČNE BESEDE

korekcijska ploskev, EGM2008, tehnika RTM, 2D bočni kontrast gostote, seizmološki podatki

ABSTRACT

The present paper describes the construction of a new vertical reference surface over northern part of Algeria based on EGM2008/RTM geoid model as support to earthquake and geodynamical applications. The residual terrain model (RTM) reduction has been applied for computing estimates of the omission error (high frequency signal) using the two-dimensional lateral density variations model derived from geological maps. The comparisons of the EGM2008-only with a set of precise GPS/levelling data shows standard deviations of 8.5cm and 5.5cm before and after fitting, respectively. Applying RTM-based omission error estimates to EGM2008 reduced its standard deviation to 2.0 cm after fitting and therefore improves the geoid modelling by almost 63.5 %. Furthermore, the EGM2008/RTM based geoid model displays an improvement in reliability than the latest local and African gravimetric geoids solutions.

In second part, a correlation study between the topographical data, 2D lateral density variations model, some gravity field parameters and seismological data, was been conducted. The results of this study show a significant correlation between geoid undulation and earthquake magnitude parameter. It is around 40% for earthquakes whose magnitude exceeds 5 on the Richer scale.

KEY WORDS

corrector surface, EGM2008, the residual terrain model (RTM), omission error, two-dimensional lateral density variations model, seismological data

1 Introduction

Algeria is one of the most seismically active areas in the Mediterranean basin capable of generating low-to high-intensity earthquakes, particularly along the boundary between the African plate to the south and the Eurasian plate to the north and therefore requires an accurate and continuous vertical reference surface for earthquake monitoring and geodynamical applications. So, the national levelling network had been established many years ago and a significant number of benchmarks have been destroyed and some of them have been exposed to unknown vertical deformation. Moreover levelling over areas with rough terrain like Algeria is very tough and time consuming. Accordingly, the problems of densifying, replacing and monitoring of the levelling network in Algeria have constrained the use of other techniques. The combination of GPS heights with geoid heights can provide an efficient alternative to derive orthometric heights. However, the crucial part of this method is the geoidal undulation, which is normally obtained with a lower accuracy than the ellipsoidal height and affects the accuracy of the orthometric height. In Algeria, the only attempt for gravimetric geoid determination has been done through the combination of global models with the few available scattered and validated gravity data from only BGI (Bureau Gravimétrique International) database. The results are not accurate enough to support the GPS levelling technique (Benahmed Daho et al., 2009). So in the framework of this work, the recommended RTM approach (Hirt et al., 2010) for modelling the high-frequency gravity field (omission error) in areas without sufficient regional gravity data coverage, like Algeria, is investigated to improve geoid information from the global geopotential model EGM2008. The combined EGM2008 Earth Gravitational Model, released by the US National Geospatial Agency (Pavlis et al., 2008), is a state-of-the-art high-degree global geopotential model (GGM) of the Earth's external gravity field. It is completed to degree and order 2159 and contains additional spherical harmonic coefficients up to degree 2190, which account for the transformation from ellipsoidal to spherical coefficients. This corresponds to a spatial resolution of 5 arc minutes or ≈ 9 km. As such, any gravity field structures at scales shorter than 5 arc minutes are not represented by the EGM2008 degree-2190 series expansion (Hirt et al., 2010). The signal not represented in the global models is commonly defined as an omission error. The height anomalies from a degree-2190 expansion may be affected by signal omission errors of several centimeters. This missing high-frequency power the gravity field can be compensated by that computed from high resolution digital elevation models using the topographic correction. The residual terrain modelling (RTM) technique is generally applied to recover the short-scale gravity field signals. The basic principle of RTM involves filtering in geometry through subtracting a smooth reference surface from a detailed Digital Elevation Model (DEM). Subsequently, the high-frequency portion of topographic masses between the Earth's surface and the reference surface is obtained. In this work, the reference surface needed for the RTM reduction has been constructed by two ways; by means of a moving average applied to the DEM based SRTM data and from DTM2006.0 spherical harmonic model of Earth's topography to degree 2160, which corresponds to the maximum degree of the global gravity field model EGM2008. A comparison was carried out between them.

The main objective of this paper is to try to construct a new vertical reference surface in the northern part of Algeria based on the EGM2008 geoid model on which we applied the RTM reduction to restore the high-frequency gravity field using the two-dimensional lateral density variations model derived from geological maps. This model has been evaluated by using a set of homogeneous and precise GPS/levelling data collected from the local GPS/Levelling surveys covering the northern part of Algeria. Additional geoid solutions have been included in this comparison in order to quantify the possible improvements over local geoid solution ;

the EGM2008-only, the last published Algerian gravimetric local geoid model GEODALG09 (Benahmed Daho et al., 2009) and the African Geoid AFRgeo2019 (Abd-Elmotaal et al., 2020).

Furthermore and considering that the geoid provides important information about the Earth's internal structure and can be used to locate the deep source of mass anomalies, a correlative study between the topographical data, 2D lateral density variations model, gravity field parameters and some seismological data, has been conducted in order to find the possible correlation between them.

2 Description of used data

Several data sets were used. They comprise a global geopotential model EGM2008, digital elevation model based on SRTM data, two-dimensional topographic mass density model and geometric geoid heights from various GPS/levelling benchmarks.

2.1 GPS/Levelling data

There are several GPS/levelling benchmarks distributed over some regions of Algeria principally in the north part of the country. For this investigation, 48 precise GPS levelled benchmarks have been used of which 31 are the first order levelling network, and the others belong to the second order levelling network. The geographical distribution of the available GPS/Levelling data is shown in Figure 1. The GPS observations were processed with the Bernese GPS software (Beutler et al., 2001) using the precise ephemerides supplied by IGS (International GNSS Service). The computed ellipsoidal heights were referred to WGS84 system and their standard deviations do not exceed 2 cm. All GPS stations have been connected to the national levelling network, which consists of orthometric heights. The connection of the most GPS stations to the levelling network has been carried out by spirit levelling. The indirect levelling was used for some GPS benchmarks located in mountainous regions in which spirit levelling would be impractical.

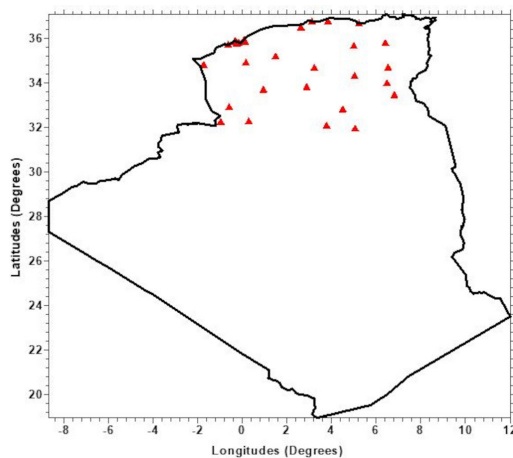


Figure 1: Geographical distribution of available GPS/Levelling stations in the north of Algeria

2.2 Geopotential model

During the last decade, the realization of the satellite gravity missions of CHAMP, GRACE and GOCE, the acquisition of new gravity data and the development of novel processing methodologies has led to

the determination of more accurate and higher resolution global geopotential models. The most revolutionary global model is the Earth Gravitational Model 2008 (EGM2008) developed by the National Geospatial-Intelligence Agency (NGA) of USA, superseding its processor EGM96 (Lemoine et al., 1998). EGM2008 combines satellite (GRACE), marine (satellite-altimetry-derived), and land gravity data to model the global gravity field with a spatial resolution of 5 by 5 arcmin. Its accuracy is largely dependent on accuracy of GRACE, marine and land gravity data and their availability (Pavlis et al., 2008, 2012).

As Global Gravity Models (GGMs) play an important role in local geoid modelling using the remove-restore technique, the first test was to validate in the area under study, the high-resolution Earth Gravitational Model EGM2008. This model was extensively evaluated over Algeria territory using the gravity data supplied by the BGI, and some GPS/levelling data. Based on used data sets, the released Earth Gravity Model EGM2008 is consistently superior to other tested geopotential models in Algeria (Benahmed Daho, 2009).

Furthermore and since the launch of the GOCE gravity field mission in March 2009, a number of global gravity field models based on GOCE data have been produced by ESA's High Level Processing Facility (HPF) and by a few other teams. Some of these models use combinations of different satellite missions, while others use only GOCE data. In this research, the following combined geopotential models EIGEN-6C4 (Förste et al., 2014) and XGM2019e (Zingerle et al., 2020), including the GOCE satellite gravity gradiometry data, have been compared with EGM2008 for different truncation degrees using the same data used in previous assessments (12472 land BGI gravity data and a set of GPS/levelling benchmarks mentioned above) in order to identify up to which degree the models provide significant results, or in other words when do they start to lose signal due to the attenuation of the gravity signal with satellite height (Rexer & al., 2013).

The combined gravity field model EIGEN-6C4 is the combined global geopotential model of GFZ Potsdam and GRGS Toulouse. EIGEN-6C4 has been generated including the satellite gravity gradiometry data of the entire GOCE mission (November 2009 till October 2013) and is of maximum spherical degree and order 2190. While the XGM2019e is a combined global gravity field model represented by spherical harmonics up to degree and order (d/o) 5399, corresponding to a spatial resolution of 2' (~4 km). As data sources, it includes the satellite model GOCO06s in the longer wavelength range up to d/o 300 combined with a ground gravity grid which also covers the shorter wavelengths. Over the oceans, gravity anomalies derived from satellite altimetry are used (DTU13 with a resolution of 1').

So, terrestrial point data (such as the gravity data and/or the GPS/Levelling derived geoid heights) are not 'compatible' with the corresponding values obtained from GGM's spherical harmonic expansion due to the limited spatial resolution that is inherent in every GGM. Therefore, some a-priori filtering model should be applied first to the terrestrial point data, before they are compared to the corresponding quantities obtained from the tested GGM. This is done in order to tune the validation data within the same spectral bandwidth provided by the GGM (Gruber, 2004).

The BGI free air gravity anomalies (Δg_{obs}) are compared with corresponding values computed from the tested geopotential models (Δg_{CGM}). Therefore and before comparing them and in order to make a fair comparison and taking into account the GGM's omission error, we will have to estimate the omitted signal of the global gravity field model by other means. The EGM2008 model was used for the estimation of the omission error. The reduced free-air gravity anomalies ($\Delta g_{obs} - \Delta g_{CGM}$) are computed for each tested geopotential model for

different truncation degrees and orders in steps of 50 starting from degree and order 2 to 2190. The most informative of these statistics is the Root Mean Square (RMS) of the difference because the mean of the differences is distorted by the exclusion of the zero-order term. The same procedure described above was applied to GPS/levelling derived geoid heights. The RMS values of the differences are shown as a function of the spherical harmonic degree. Figures 2 and 3 visualise the evaluation results for GPS/levelling derived geoid heights and the BGI free-air gravity anomalies differences, respectively. Considering Figure 2, it can be seen that all tested GGMs show comparable performance up to degree 50 and that the GOCE-based models provide slightly improved information on the bands ~250 to ~900 in comparison to the EGM2008 model. The improvement is about the 1 to 4 mm that remains no significant. Furthermore, we can see a slowly progressing signal loss for XGM2019e geopotential model from degree 950 compared to EGM2008 model taken as reference. According to our numerical results, the EGM2008 model fits better the BGI free-air gravity anomalies and remains again superior to all tested geopotential models. Further improvement should be expected with upcoming releases of GOCE-based GGMs. The future release GOCE-based GGMs will considerably improve the determination of geoid height in the areas where the EGM2008 performs poorly such as Africa, South America and South-East Asia.

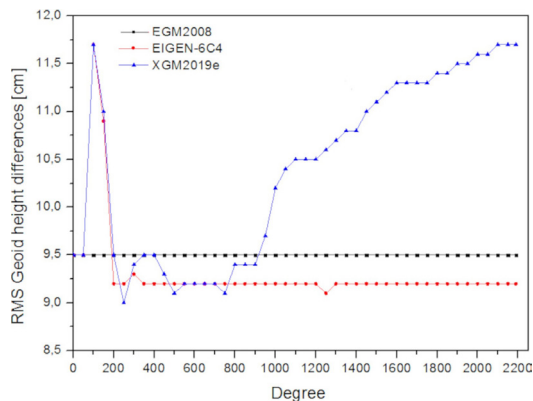


Figure 2: RMS of the geoid heights differences as a function of the GGM used in spectral window 2 to 2190.

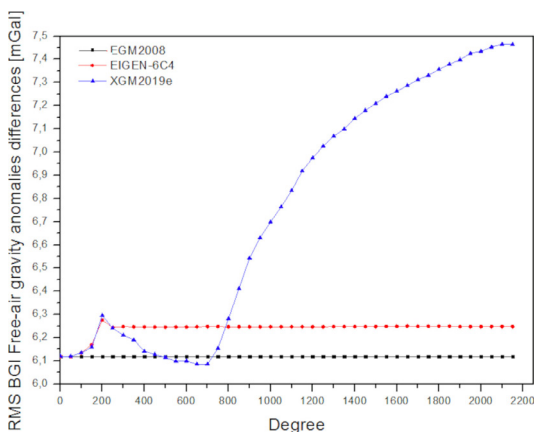


Figure 3: RMS of the free-air gravity anomalies differences as a function of the GGM used in spectral window 2 to 2190.

2.3 Digital Elevation Model

In gravity reduction to correct for terrain effects and in gravity interpolation/prediction to smooth the gravity field, Digital Terrain Model (topography and bathymetry data) of the region is required. The recently available SRTM digital elevation data set opens new perspectives for the regional or local determination of the gravity field. However, due to the lack of any high resolution photogrammetric based DEM in Algeria, a new DEM model was constructed with a resolution of 15" by use of recent 3" (-100 m) high resolution SRTM. The output grid size is limited to $15^\circ \leq \varphi \leq 41^\circ$ and $-15^\circ \leq \lambda \leq 15^\circ$. On the Mediterranean Sea, this DEM was completed from the bathymetric ETOPO2 model.

Benahmed Daho et al. (2009) have tested of the suitability of the constructed DEM by comparing it to the heights of the land BGI gravity data. In this comparison, the GTOPO DEM with a resolution of 30" x 30" used in previous gravimetric geoid computations in Algeria, was included (Benahmed Daho, 2000). According the accuracy of SRTM data, 83.7 % (82.9 % respectively for GTOPO) of the covered area is in agreement with BGI heights data better than ±16 meters (the given SRTM vertical accuracy on the 90 % level). In most points and for both tested DEM, the differences are smaller than 10 meters. There are very few stations where really large differences occur. Usually they are located in areas where there is a SRTM data lack nearby or in the southern part of the country (Benahmed Daho et al., 2009).

In the framework of this study, the DEMs described above were also compared in absolute sense with a number of precise levelling benchmarks (the previous GPS/Levelling data set). In practice, the systematic errors in DEMs solutions and datum inconsistencies inherent in the different height data sets may be approximated by different kinds of functions in order to fit the interpolated heights from DEM (H_{DEM}) to levelled orthometric heights ($H_{Levelling}$) through an integrated least squares (LS) adjustment. In order to minimise them, several models can be used ranging from a simple linear regression to a more complicated seven-parameter similarity transformation model. In this paper, we have tested various models (Third degree polynomial, four, five and seven parameter models) in order to determine the possible an improved transformation model that can describe more effectively the general trend of the discrepancies between the interpolated heights (H_{DEM}) and the levelled orthometric heights ($H_{Levelling}$). According to numerical results, the four-parameter model fits best with minimum standard deviation versus orthometric heights. Such a comparison is based on the following model (Heiskanen and Moritz, 1967):

$$H_{Levelling} - H_{DEM} = X_0 + X_1 \text{Cos}(\Phi) \cdot \text{Cos}(\lambda) + X_2 \text{Cos}(\Phi) \cdot \text{Sin}(\lambda) + X_3 \text{Sin}(\Phi) + V_i \quad (1)$$

where Φ and λ are the horizontal geodetic coordinates of the levelling benchmark. The vector of unknown parameters is solved by minimizing the quantity $\mathbf{v}^T \mathbf{v}$. The adjusted values for the residuals \mathbf{v}_i give a realistic picture of the level of absolute agreement between the DEMs solutions and levelled heights. The Table 1 summarises the statistics of the differences between the tested DEMs and GPS/Levelling ones at benchmarks before and after fitting out the systematic biases and tilts using the four-parameter model transformation. These results show that the DEM based on SRTM performs better with respect to GPS/levelling data compared to GTOPO DEM. The use of the fitting procedure does not improve the results significantly. It is important to mention here that the estimated accuracy for the DEM based on SRTM data in Algeria is about 8 m, which is much better than the overall global estimation of accuracy for this model (16 m). In conclusion, the comparisons of the different DEM with the BGI and Levelled heights show that the

constructed DEM based on SRTM data agrees considerably better than the GTOPO one. Nevertheless, further comparisons remain necessary to test again the quality of DEM based on SRTM in Algeria.

Table 1: Comparison of tested DEMs solutions with GPS/Levelling heights before and after the bias and tilt fitting (Unit: [m]).

	Differences	Min	Max	Mean	Sd.
Before	SRTM	-6.761	34.854	4.189	8.157
	GTOPO	-50.650	68.265	3.081	15.396
After	SRTM	-9.954	30.618	0.000	7.619
	GTOPO	-43.551	64.757	0.000	14.691

2.4 Two-dimensional topographic mass density model

In this research, the geological map approach was adopted to generate the lateral mass density variation model. The establishment of the national geological map is one of mission of the National Geological Survey. This service has published the Geological Map of Algeria in 1:2 000 000 scale, which displays bedrock formations at or near the Earth's surface. The bedrock units are grouped according to composition and geological age. The northern part of Algeria is composed of 33 fundamental units.

In general, the northwest consists of the following geological units: Quaternary, Pliocene, Miocene, Lower Cretaceous, Middle Jurassic, Upper Jurassic, whose density varies between 2.12 and 2.65 g/cm³. The northeast consists mainly of Kabyle basement, Quaternary, Cenozoic-granite, Miocene, Oligo-Miocene, and Eocene, whose density varies between 1.95 and 2.6 g/cm³. The north-central part consists of volcanism, Quaternary, Miocene, Oligocene, Neogene, and Cretaceous whose density variation is almost identical to that of the Northeast part, the density varies from 1.95 to 2.65 g/cm³. Finally, the remainder (south part) is characterized by the presence of the following units: quaternary, dune sand, Pleistocene Pliocene Neogene and whose density is between 1.82 and 2.4 g/cm³.

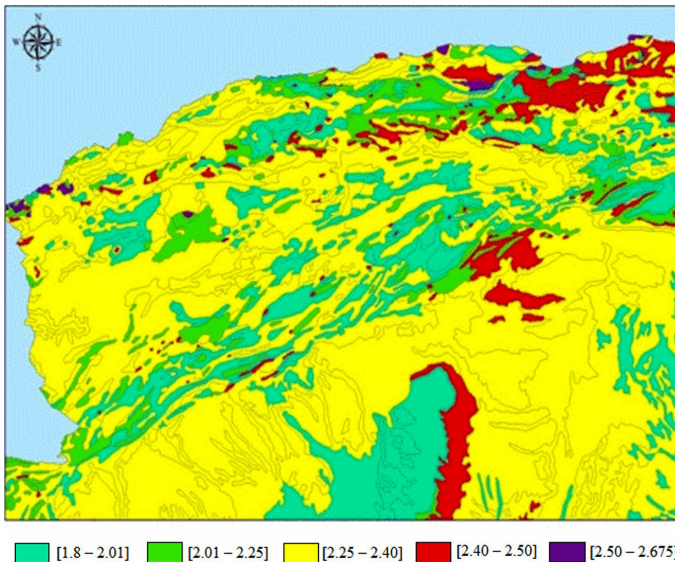


Figure 4: Map of density model on the northern part of Algeria [Unity: g/cm³]

To generate the two-dimensional topographical mass density map from the digital geologic map under ArcGIS, we have used the legend information published by the website (www.mineralinfo.org), where each geological unit includes a set of rocks of varying densities. The density of the geological unit will then be the mean value of all densities of the rocks that compose it. The results are displayed in Figure 4, which shows large contrasts in the density variation of rocks with maximum and minimum values of 2.675 and 1.82 g/cm³, respectively and proves that the constant density 2.67 g/cm³ used in the gravimetric geoid determination by Stokes formula can constitute a source of errors limiting the accuracy of the geoid.

3 EGM2008/RTM based geoid model determination

The computation of the effects of the topography according to the RTM reduction modelling method is based on previous 15" x 15" digital terrain model from SRTM which was used up to a distance of 100 km. The reference surface of 5' x 5' needed for the RTM reduction has been obtained by two ways; by means of a moving average applied to the detailed one and from DTM2006.0 spherical harmonic model of Earth's topography developed to spherical harmonic degree 2160. The transformation of RTM elevations to residual height anomalies is accomplished using forward-modelling gravitational potential formulas for prisms (Hirt et al., 2010).

A first test of the suitability of constructed DEM reference surfaces for this purpose was performed by comparing them with the heights of 12472 land BGI gravity data covering the territory of Algeria. These measurements dated back to 1950 and were collected for exploration geophysical, rather than physical-geodetic purposes. All gravity data was then referred to IGSN71 and the GSR80 system was used for normal gravity. Many systematic and random errors exist in these data. Hence, editing the data is critical for accurate geoid computation. All data have been checked carefully to remove repeated observations, and validated for gross errors by applying cross-validation technique. There are some problems in the data in the mountainous regions. The differences may be a combination of errors in the gravity observation and location. Of all the error sources, the errors in recorded height for the gravity measurements are likely the major contributor. The elevations of the gravity survey are mostly read from a topography map, a common practice before the advent of GPS. This practice may introduce height errors in the range of meters to hundreds of meters. Figure 5 shows the geographical distribution of the BGI land gravity data. Table 2 summarises the statistics in meter of the differences between constructed DEM reference surfaces and BGI heights data. According the accuracy of SRTM data, 70.4 % (68.5 % respectively for DEM2006 spherical harmonic model of Earth's topography) of the covered area is in agreement with BGI heights data better than ±16 meters (the given SRTM vertical accuracy on the 90 % level). The comparisons of the different reference surface DEM with the BGI heights show that the constructed DEM based on SRTM data directly agrees better than the DEM2006 one. Nevertheless, further comparisons remain necessary to test again the quality of SRTM in Algeria.

Table 2: Statistics of the differences between BGI heights data and constructed DEM reference surfaces

Reference surface	Min	Max	Mean	Sd.
From SRTM	-558.833	429.806	7.172	<u>39.784</u>
From DTM2006	-548.965	511.737	6.398	40.558

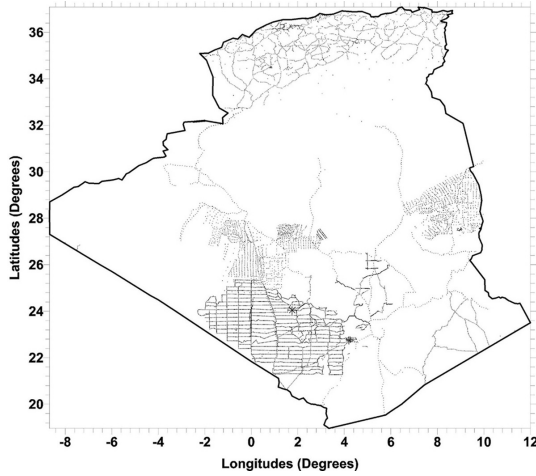


Figure 5: Geographical distribution of the BGI land gravity data.

The terrain corrections were computed in two steps on a 5'x 5' grid covering the area bounded by the limits $19^\circ \leq \varphi \leq 39^\circ$; and $-9.5^\circ \leq \lambda \leq 12^\circ$ using the constructed DEM reference surfaces. Firstly, the topographic correction to be applied to EGM2008 based geoid model according to the RTM reduction modelling method has been computed with a constant density (2.67 g/cm^3) using the Fortran program TC of the GRAVSOFIT package and the strict numerical integration without any approximation (Tscherning et al., 1992).

In the next step, the above EGM2008 based geoid model was corrected by considering the real topographical density distribution in order to improve its accuracy in mountainous areas. The knowledge of real crust density is required in terrain correction in order to remove effectively and rigorously all the masses above the geoid. The constant density is often used in practice instead of actual crust density because of lack of actual bedrock density information. This introduces errors on the reduced gravity anomalies and, consequently, on the geoid of the decimetre level in mountainous areas. However, two-dimensional digital density models (DDMs) are now becoming available, but a three-dimensional model is required to represent a real topographical density distribution and reflects the behaviour of bedrock with depth. These density models should be incorporated in the terrain correction (TC) computation.

For the computation of the lateral density effect on the geoid, we have used, in the framework of this research, the practical computational formula developed by Sjöberg in which the total effect of lateral density variations on geoid composed of the direct, downward continuation and indirect effects is represented as a simple correction proportional to the lateral density variation and the square of elevation of the computation point. The formula can be applied easily as an additive correction for geoid heights for any geoid model. This effect on the geoid due to lateral density anomaly can be approximated by (Sjöberg, 2004):

$$\delta N_{comb}^t \approx -\frac{2\pi\mu}{\gamma_0} H^2 \text{ avec } \mu = G\rho \quad (2)$$

Here G is the gravitational constant, ρ is the topographic mass density at Latitude ϕ and Longitude λ , H and γ_0 are, respectively, height and normal gravity of the computation point.

However, if the density of the topography at the computation point is $\rho = \rho_0 + \Delta\rho$ where ρ_0 the standard density (2.67 g/cm^3) and $\Delta\rho = \Delta\rho(\phi, \lambda)$ is the lateral density anomaly, then the total effect of $\Delta\rho$ on the geoid height became (Sjöberg, 2004):

$$\delta N_{\Delta\rho} \approx -\frac{2\pi G \Delta\rho}{\gamma_0} H^2 \tag{3}$$

The total effect on the geoid caused by the lateral variation of topographic mass density was computed on a grid covering only the northern part of Algeria between the limits $[31^\circ, 37^\circ]$ in Latitude and $[-2^\circ, 7^\circ]$ in Longitude using the formula 3, with a spacing of $5' \times 5'$ (same resolution of the EGM2008 based geoid model). Figure 6 shows the effect of the density variation model on the geoid. It varies between 0 and 13 cm and remains significant for a centimeter determination of the geoid. Figure 7 represents a map of the EGM2008/RTM-DTM2006 geoid solution in Algeria corrected of effect of the real density variation model.

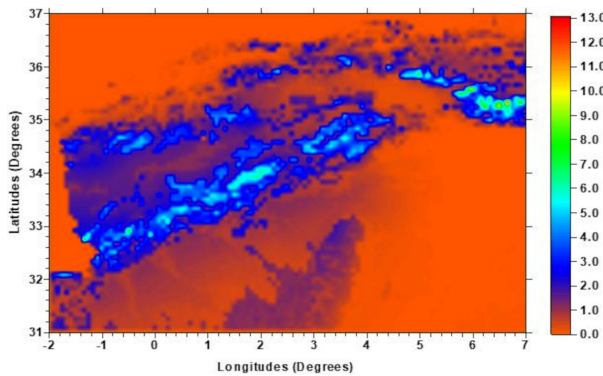


Figure 6: Effect on the geoid undulation resulting from the use of the real topographic mass density model [Unit: cm].

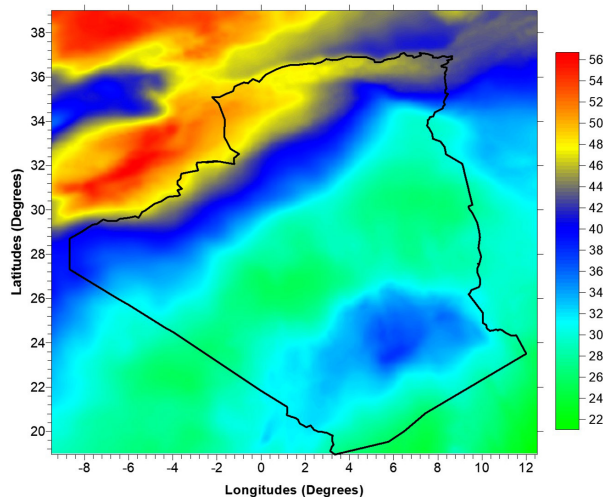


Figure 7: Map of the Algerian geoid model based on EGM2008/RTM-DTM2006 corrected of effect of the lateral density model [Unit: m]

4 Comparison of the EGM2008/RTM based geoid model with GPS/Levelling data

The comparison between gravimetrically derived and the highest quality and most reliable GPS/Levelling derived geoid undulations, over a network of spirit levelling benchmarks has been a standard routine task for testing the performance of global spherical harmonic models for the Earth's gravity field and the quality of local/regional gravimetric geoid solutions and estimating spatial parametric models for optimal orthometric height transformation between geoid surface and levelling datum surface.

For our study, different geoid solutions were compared with 48 co-located GPS/levelling geoid undulations in absolute and relative sense. Two geoid models based on EGM2008/RTM omission error correction using the previous reference surfaces called EGM08/RTM-SRTM and EGM08/RTM-DTM2006, the EGM2008-only, the last published Algerian gravimetric local geoid model GEODALG09 and the African Geoid AFRgeo2019. The GPS/levelling geoidal height $N_i^{\text{GPS/Levelling}}$ is the result of the difference between the ellipsoidal height from GPS observation and the orthometric one obtained by spirit levelling and gravity information.

4.1 Absolute evaluation

All these 48 GPS levelled stations are used as benchmarks in a least-squares adjustment using the Collocation approach in iterative process to solve for the unknown parameters of the pre-selected corrector surface. Figure 8 represents the general adjustment procedure using the collocation approach. For this purpose, a FORTRAN program has been developed for combined adjustment between the geoid model and GPS/Levelling data using the collocation approach.

To minimise the datum inconsistencies between the available height data, several tests by using different parametric models ranging from a simple linear regression to a more complicated seven-parameter similarity transformation model, were conducted to find an adequate functional representation of the correction that should be applied to the geoid model and that can describe more effectively the general trend of the discrepancies between the GPS/Levelling and the geoid model. Such a comparison was based on the following Collocation model:

$$N_i^{\text{GPS/Levelling}} - N_i = h_i - H_i - N_i = \mathbf{a}_i^T \mathbf{X} + \mathbf{s}_i + \mathbf{n}_i \quad (4)$$

Where h_i , H_i and N_i denote the available observed values for the GPS, orthometric and geoid height respectively. The quantity $\mathbf{a}_i^T \mathbf{X}$ representing the parametric model supposed to describe the systematic errors and datum inconsistencies inherent in the different height data sets.

From a statistical point of view, the signal \mathbf{s}_i represents the correlated part of total random error, while \mathbf{n}_i is the uncorrelated part. From a geodetic point of view, \mathbf{s}_i is mainly due to errors in gravity data which of their integration cause correlation between gravimetric geoid height, GPS or Levelling errors are partly correlated. \mathbf{n}_i contains errors due to heights of GPS antenna, unexpected vertical motion of benchmarks, etc. The quantity \mathbf{s}_i is used together with the parametric model to correct the geoid model and to adapt it to a vertical reference surface obtained by use of the set of GPS levelled benchmarks.

According to numerical results, the linear trend model fits best with minimum standard deviation versus GPS/levelling data in study area. Such model is based on:

$$\mathbf{a}_i = \begin{pmatrix} \text{Cos}(\Phi_i)(\lambda_i - \lambda_0) \\ \Phi_i - \Phi_0 \\ \mathbf{1} \end{pmatrix}, \text{ and } \mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \mathbf{X}_3 \end{pmatrix} \tag{5}$$

With (Φ_i, λ_i) and (Φ_0, λ_0) are, respectively, the horizontal geodetic coordinates of the network points and the reference benchmark, \mathbf{X}_1 and \mathbf{X}_2 are the slopes in Est-West and North-South directions, respectively, and \mathbf{X}_3 is the vertical bias.

The comparison of the local analytical covariance functions with the empirical covariance one shows that the Gauss model is optimum in the computation area. The agreement is optimum between them until a spherical distance of around 0.23° (~ 23 km).

The statistics of the differences at benchmarks before and after fitting out the systematic biases and tilts between the geoid models and the GPS/levelling data using the linear trend parameter model are summarised in Table 3. Some conclusions can be drawn:

- Based on the GPS levelled data set used in this evaluation, the maximum effect of the lateral density variation model on these benchmarks is about of 5cm because our GPS/levelling stations are located in moderate heights areas with a maximum height of 1200m (see Figure 9).
- In our test area, the comparisons of the EGM2008 only with the GPS/levelling data showed standard deviations of 8.5 cm and 5.5 cm before and after fitting using the linear trend as corrector surface, respectively. Augmentation with RTM omission error estimates reduced its standard deviation to 2.0 cm after fitting. These results demonstrate that applying RTM omission error estimates to EGM2008 based geoid model improves the geoid modelling by almost 63.5 %.
- The EGM2008/ RTM-DTM2006 based geoid model shows an improvement in reliability than the latest local and African tested gravimetric geoids models. This can be mainly attributed to the more accurate long-wavelength information contained in EGM2008 in comparison to EIGEN-GL04C and EGM96 geopotential model used, respectively, in GEOIDALG09 and AFRgeo2019 solutions.

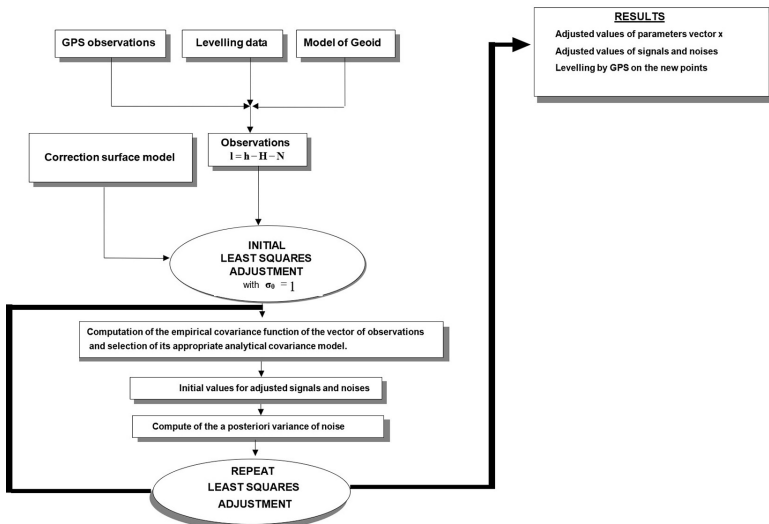


Figure 8: Flowchart of the general adjustment procedure (Collocation approach)

Table 3: Comparison of tested geoid models with GPS/Levelling undulation before and after the bias and tilt fitting (Unit: [m])

	Differences	Min	Max	Mean	RMS
Before	EGM08/RTM-SRTM	-0.534	-0.129	-0.361	0.085
	EGM08/RTM-DTM2006	-0.557	-0.136	0.346	0.093
	EGM2008-only	-0.527	-0.123	-0.336	0.086
	GEOIDALG09	-0.213	1.097	0.428	0.274
	AFRgeo2019	-1.643	-0.698	-1.101	0.187
After	EGM08/RTM-SRTM	-0.099	0.099	0.000	0.042
	EGM08/RTM-DTM2006	-0.030	0.067	0.000	0.020
	EGM2008-only	-0.127	0.116	0.000	0.055
	GEOIDALG09	-0.680	0.623	0.000	0.225
	AFRgeo2019	-0.293	0.200	0.000	0.111

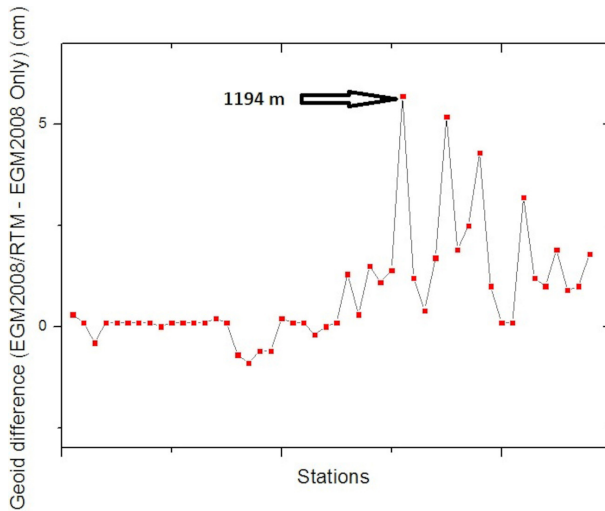


Figure 9: Geoid differences in benchmarks between EGM2008/ RTM-DTM2006 and EGM2008-only based geoids models

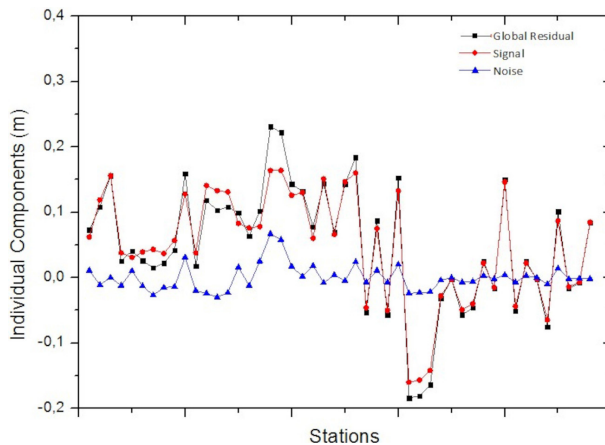


Figure 10: Individual components global residuals, signals and noises in benchmarks.

The Figure 10 illustrates the individual components in benchmarks of the global residual, the signals and the noises. The signals are dominated by errors in the geoid due to the poor and sparse gravity data included in EGM2008 global geopotential model, while the noise level is an indication of the presence of possible errors in our national vertical Datum.

4.2 Relative Evaluation

In order to show the real potential of a geoid model, the above geoid solutions were evaluated also in the relative sense by using the previous 48 GPS/Levelling data. This type of assessment is more informative to the GPS user than the absolute evaluation since most GPS surveys are performed in the relative mode and because the majority of the systematic errors are canceled or reduced eliminated through the differencing of observations. The computations were carried out using the FORTRAN program developed by Featherstone (Featherstone, 2001).

The Figure 11 shows the empirical functions of relative accuracy for tested geoid models. These functions were computed using the following formula:

$$C(\Psi) = \frac{1}{M} \sum \sigma_i \tag{6}$$

where M is the number of combinations, σ_i is the relative accuracy in PPM (Parts Per million) for each distance Ψ . The summation was made for all the combinations of the data points Q_i and Q_j where the interstation distances in the range between $\left(\Psi - \frac{\Delta\Psi}{2}\right)$ and $\left(\Psi + \frac{\Delta\Psi}{2}\right)$, denotes the sampling interval size. We can see that the geoids models EGM08/RTM-SRTM, EGM08/RTM-DTM2006 and the EGM2008-only give almost the same results in the relative sense with a slight superiority for the EGM2008-only for the short distances. The relative agreement ranges from 5 to 2 ppm over short distances of about 20 to 50km and less than to one ppm for baselines of 100 to over 800 km.

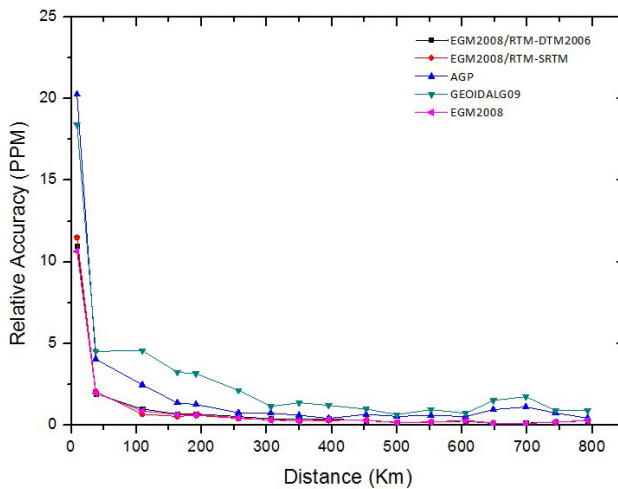


Figure 11: Empirical function of relative accuracy for all tested geoid models (sampling interval size 50km).

5 Altimetric auscultation of a six-storage reservoir

In order to test the ability of GPS for height determination, special GPS campaign was carried out for Altimetric auscultation of a six oil storage reservoir located in industrial zone in west of Algeria. A total, 24 new GPS stations were established with four benchmarks for each reservoir. All GPS stations have been connected to the national height system through spirit levelling, which consists of orthometric heights. The observed orthometric heights in benchmarks have been compared with the corresponding adjusted ones derived from the combination of ellipsoidal heights and the corrected geoid undulation based in EGM2008/RTM-DTM2006 geoid model. The statistics of these differences are summarized in Table 4.

Their analysis permits to conclude that the orthometric heights on benchmarks can be deducted from the GPS observations with a precision better than 1cm except for the B082 benchmark (suspect GPS measure). 96 % and 71 % of the differences are smaller than 1cm and 5mm, respectively. This precision remains sufficient for geodynamical applications.

Table 4: Discrepancies, in benchmarks, between the observed heights and computed ones

Benchmarks	Observed height (m)	Computed height (m)	Discrepancies (cm)
B031	8.108	8.105	-0.3
B032	7.982	7.979	-0.3
B033	8.254	8.249	-0.5
B034	8.215	8.219	0.4
B041	8.038	8.042	0.4
B042	8.032	8.040	0.8
B043	8.184	8.191	0.7
B044	8.139	8.147	0.8
B051	8.040	8.032	-0.8
B052	8.090	8.091	0.1
B053	8.317	8.316	-0.1
B054	8.099	8.093	-0.6
B061	8.158	8.156	-0.2
B062	7.919	7.915	-0.4
B063	8.280	8.281	0.1
B064	8.251	8.248	-0.3
B071	8.150	8.155	0.5
B072	8.054	8.053	-0.1
B073	8.243	8.243	0.0
B074	8.140	8.140	0.0
B081	8.033	8.027	-0.6
B082	8.215	8.131	-8.4
B083	8.380	8.376	-0.4
B084	8.247	8.252	0.5

6 The new vertical reference surface over north part of Algeria

In order to construct a new vertical reference surface over northern part of Algeria, we try to combine the best EGM08/RTM-DTM2006 based geoid model with the previous 48GPS/levelling data by using a corrector surface concept. By applying the linear trend parameter fitting approach and interpolated values of the signal, the 5'x 5' continuous surface between the limits $31^{\circ} \leq \varphi \leq 37^{\circ}$ and $-2^{\circ} \leq \lambda \leq 9^{\circ}$ is generated from the discrete GPS/levelling data using different prediction techniques whose the choice is crucial for the result. In this work, and for creating a continuous surface, we have tested in addition to Least Square Collocation technique, several others prediction techniques. Among them, the Minimum curvature, Inverse distance, and Kriging, all implemented in Surfer Software. For the Collocation technique, the prediction formula is given by:

$$\hat{s} = C_{st} (C_{tt} + C_{nn})^{-1} L \tag{7}$$

Where C_{st} is the covariance vector between the signal S and vector of observations L , $\bar{C} = C_{tt} + C_{nn}$ is the sum of the covariance matrices of the measurements and the random measuring errors, taken as diagonal and constant: $C_{nn} = \sigma_0^2 I_{tt}$ where I is identity matrix.

The first step in the use the Collocation method is the estimation of the empirical covariance function and subsequent selection of the best analytical representation. In this work, three different analytical covariance functions broadly used in physical geodesy were tested for the selection the appropriate analytic covariance model: the simple exponential function, Hirvonen model and the second-order Markov. The free parameters in these models are variance C_0 of data and correlation length ζ . These parameters are estimated by fitting these models to a number of empirical covariance values using the least square adjustment.

The empirical covariance function of the corrector surface in benchmarks was computed using the following formula:

$$C_{ss}(\Psi) = \frac{1}{M} \sum s_i s_j \tag{8}$$

Where M is the number of combinations, Ψ is the spherical distance between Q_i and Q_j and S_i is the total correction value in benchmarks should be applied to the geoid. The summation was made for all the combinations of the data points Q_i and Q_j whose distance was comprised between $\left(\Psi - \frac{\Delta\Psi}{2} \right)$ and $\left(\Psi + \frac{\Delta\Psi}{2} \right)$, denotes the sampling interval size.

According our numerical tests, the Collocation by Least Square technique gives the smallest statistics and was used for creating continuous corrector surface in this study area. The Figure 12 shows the map of a new vertical reference surface aver the north part of Algeria generated using the Collocation method from the discrete GPS/levelling data and the corresponding geoid.

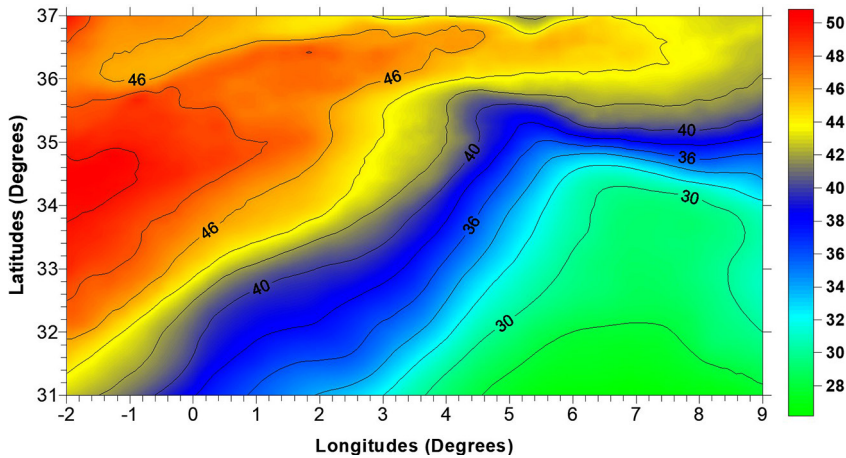


Figure 12: Map of a new vertical reference surface for the north part of Algeria [Unit: m]

7 Correlative study between gravity field parameters and seismological data

The geoid is a fundamental surface that defines the shape of the Earth. It can be described as an equipotential surface of the Earth's gravitational field, which is optimally agrees with the mean surface of the oceans. This level surface is considered an indispensable tool in all cartographic, civil engineering, oceanographic and navigational applications. Until the late 1980s, the geoid remained an object of scientific study. It was mainly used for research into vertical references and mean sea level. With the advent of space-based positioning techniques, and in particular the rapid development of the GPS system, the situation has changed radically. The geoid has become indispensable for converting ellipsoidal heights into heights relative of mean sea level. Moreover, the geoid's value is indisputable, both for geodetic applications and in fields of geophysics and oceanography. The study of deviations between mean sea level and the geoid - in other words, the dynamic topography of the sea - provides valuable information about the phenomena responsible for these deviations.

Geoid signals can provide important information about the distribution of mass in the Earth's interior. Geoid undulations have been used to study the viscosity structure of the mantle.

In this second part, a correlation study between the topographical data, 2D lateral density variations model, gravity field parameters (Geoid height and gravity anomaly) and some seismological data, was been conducted in order to find the possible correlation between them.

7.1 Description of used data

For this study, we have used a set of 1214 seismic point measurements recorded over the period from 25/10/2004 to November 2023. Each record containing the following attributes: (Date & Time, Latitude, Longitude, Depth, Depth type, Magnitude type, Magnitude, Region Name, Last update). These data were supplied by the Euro-Mediterranean Seismological Center (CSEM), which covers the northern part of Algeria and part of the Mediterranean Sea. For this data set, the magnitude varies between 1.8 and 5.6. Figure 13 shows the earthquakes distribution in northern part Algeria.

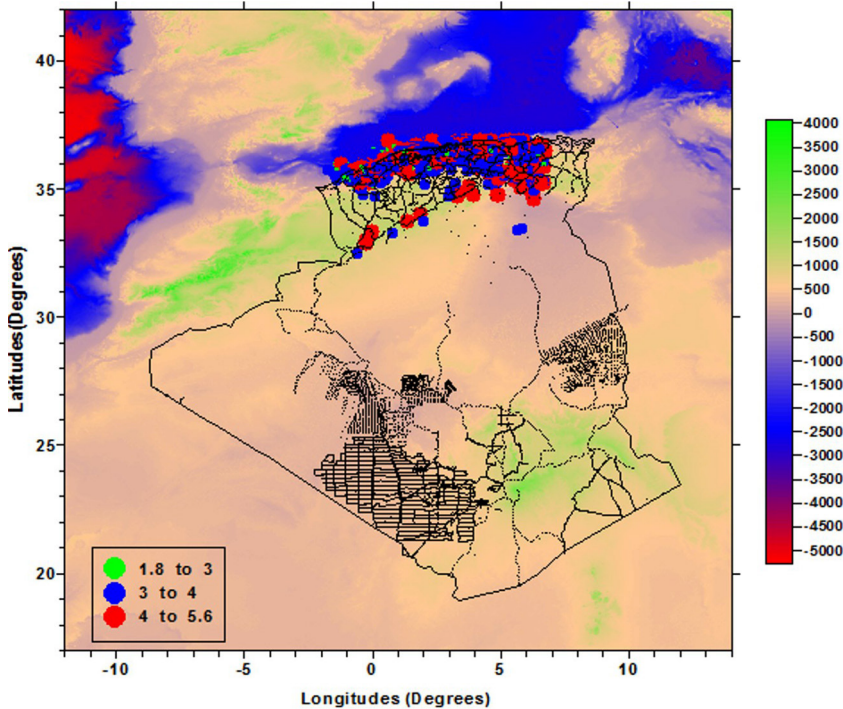


Figure 13: Earthquakes distribution in northern part Algeria [Unit: m]

The topographic height, geoid undulation and the lateral mass density for each epicentre are interpolated from the previous DEM based SRTM, EGM2008/RTM-DTM2006 based geoid model and two-dimensional lateral topographic mass density model, respectively. An average density of 1.027 g/cm^3 is considered for all localized earthquakes in the Mediterranean Sea.

So, the free-air gravity anomaly at each epicentre is estimated from the gravity database composed of 12472 previous described BGI gravity anomalies on land and 52714 altimetry-derived gravity anomalies in the Mediterranean Sea. In the Mediterranean Sea, the model DTU13 global marine gravity field (Andersen et al., 2014), with a resolution of 1 arc-minute and covering all marine study area, was used.

However, in order to improve interpolation quality, the remove-restore technique was used. Before the prediction, the free-air gravity anomalies have to be reduced by a geopotential model during the remove step. The spherical harmonic coefficient set EGM2008 complete to degree 2159 and order 2190 has been used to remove the long wavelength component of gravity data. The reduced data after reduction are significantly smoother than the original data. The reduced free-air gravity anomalies standard deviation was reduced from 34.867 mGal to 5.315 mGal (- 85% of reduction).

The prediction of the gravity anomalies at each epicenter was computed from the 5 closest points in each quadrant around the prediction point, using a second-order Markov model covariance function fitted to local data (correlation length $X_{1/2} = 15 \text{ km}$), and taking into account individual variance of the gravity data. The maximum value of the estimated formal error, which is defined by this covariance function

and by the assigned error on the gravity anomalies (2 mGals in our case) is 6.5 mGals on the area where there is no data, and 0.8 mGals along the gravimetric survey, while the R.M.S. is 1.4 mGals. The final free-air gravity anomaly at each epicentre was obtained by adding the EGM2008 contribution. Table 5 and Figure 14 show, respectively, the statistics and histograms of used data.

Table 5: Statistics of used data

Parameters	Min	Max	Mean	RMS
Height H(m)	-2744.065	1759.434	162.639	852.707
Geoid height N(m)	30.973	49.663	46.100	2.497
Free-air gravity anomaly $\Delta g(\text{mGal})$	-96.062	143.427	31.895	40.953
Magnitude M (Richter)	1.8	5.6	3.5	0.6
Mass density RHO (g/cm^3)	1.03	2.83	1.98	0.56

7.2 Results & discussion

Table 6 summarises the correlation matrix between integrated parameters used in this study. The coefficients are also compared with their critical values to test the significance of the estimated correlation values. Based on the table of critical values of Bravais-Pearson (bilateral probabilities) for correlation with significance threshold ($\alpha = 5\%$), we have obtained the critical value for all parameters. Some conclusions can be drawn:

- There is a strong correlation between topographic height and mass density.
- There is a moderate correlation between topographic height and free-air gravity anomaly.
- There is a significant negative correlation between geoid undulation and earthquake magnitude parameter. It is around 40% for earthquakes whose magnitude exceeds 5 on the Richer scale.
- There is a remarkable positive and significant correlation between free-air gravity anomaly Δg and geoid undulation.

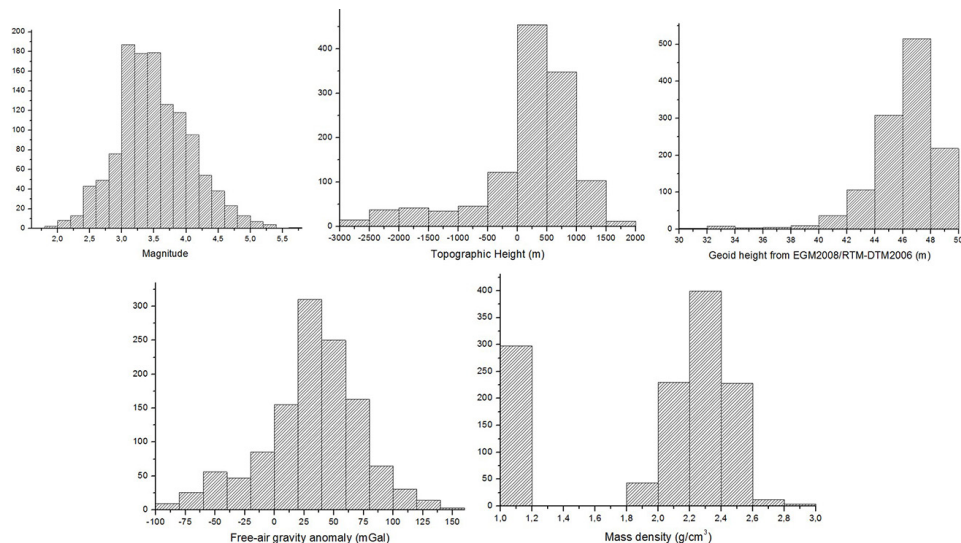


Figure 14: Histograms of used data

Table 6: Correlation matrix between integrated parameters

Variables	H	N	Δg	M	ρ
H (Topographic Height)	1.00	-0.05	0.60	0.13	0.74
N (Geoid undulation)	-0.05	1.00	0.40	-0.15	0.03
Δg (free-air gravity anomaly)	0.60	0.40	1.00	-0.03	0.43
M (Earthquake magnitude)	0.13	-0.15	-0.03	1.00	0.06
ρ (Topographic mass density)	0.74	0.03	0.43	0.06	1.00

8 Conclusion

The main purpose of this work is to investigate to improve the EGM2008 based geoid model using the RTM approach for the construction of a new vertical reference surface over northern part of Algeria based on the corrective surface concept. This surface should be convenient in engineering and for some geodynamical applications like earthquake deformation assessment. The use a corrector surface model in Least Squares adjustment problems of linked GPS/levelling/geoid vertical networks, can to reduce the major part of the discrepancies usually attributed to long wavelengths errors in the gravimetric geoid, the systematic errors and datum inconsistencies inherent in the different height data sets. Several models have been tested in order to select in the experimental area the best model using the collocation approach.

For this investigation, the global model EGM2008 in which we applied RTM reduction to restore the high-frequency gravity field using the 2D lateral density variations model and a homogenous and precise GPS/levelling network, were used. The reference surface of 5' x 5' needed for the RTM reduction has been constructed by two ways; from SRTM data directly and from DTM2006.0 spherical harmonic model of Earth's topography.

According to our numerical results, this study shows that the effect on the EGM2008 based geoid model using the 2D lateral density variations model ranges from 0 to 13cm and remains significant for a precise geoid determination with centimeter level. In our test area, the comparisons of the EGM2008-only with 48 co-located GPS/levelling geoid undulations shows standard deviations of 8.5 cm and 5.5 cm before and after fitting using the linear trend as corrector surface, respectively. Augmentation with RTM omission error estimates reduced its standard deviation to 2.0 cm after fitting. These results demonstrate that applying RTM omission error estimates to EGM2008 based geoid model improves the geoid modelling by almost 63.5%. The EGM2008/RTM based geoid model shows an improvement in precision and reliability than the latest local and African tested geoids models.

Furthermore, the analysis of the results shows that the signals in benchmarks are dominated by errors in the geoid due to the poor gravimetry, while the noise level indicates of the presence of possible errors in our national vertical datum.

In addition, the correlation study conducted between the topographical data, 2D lateral density variations model, gravity field parameters and some seismological data, shows a remarkable correlation of around 40% between geoid heights and the magnitude exceeding 5 on the Richter scale.

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