NATANČNOST VIŠIN REPERJEV GLEDE NA NAČIN IZRAVNAVE

THE IMPACT OF THE ADJUSTMENT CONCEPT ON THE BENCHMARKS' HEIGHT ACCURACY

Blaženka Bukač, Ivan Razumović, Nevio Rožić

UDK: 528.38:519.654
Klasifikacija prispevka po COBISS.SI: 1.01
Prispelo: 27. 2. 2024
Sprejeto: 21. 10. 2024

DOI: 10.15292/geodetski-vestnik.2024.04.419-431
SCIENTIFIC ARTICLE
Received: 27. 2. 2024
Accepted: 21. 10. 2024

IZVLEČEK

Nacionalni višinski referenčni sistemi temeljijo na nivelmanskih mrežah, ki so razvrščene v redove glede na njihovo natančnost. Končne višine reperjev v višinskih mrežah običajno pridobimo s posredno izravnavo meritev po metodi najmanjših kvadratov. Pogostokrat se iz pragmatičnih razlogov zanemari korelacija med opazovanji in korelacija med višinami danih reperjev. Prav tako se za višine danih reperjev običajno predpostavi, da so brez napak, čeprav imajo v resnici določeno stopnjo natančnosti. V članku je predstavljena alternativna realizacija višinskega referenčnega sistema HVRS71 za Republiko Hrvaško. Obseg podatkov je omejen na nivelmanske mreže 1. in 2. reda. Alternativna realizacija je izvedena z drugačnim konceptom prilagajanja, pri katerem so upoštevane korelacije med opazovanji zaradi uvedbe korelacije višin danih reperjev. Ta metodologija omogoča tudi prenos natančnosti absolutnih višin danih reperjev z nivelmanske mreže višjega reda na natančnost absolutnih višin reperjev nivelmanskih mrež nižjega reda. Tako se doseže objektivna, logična in dejanska porazdelitev natančnosti višin reperjev vzdolž obsega referenčnega sistema višin. Rezultati določajo, koliko sprememba koncepta prilagajanja nivelmanske mreže vpliva na kakovost višinskega referenčnega sistema.

ABSTRACT

National height reference coordinate systems are based on levelling networks classified into orders, according to their accuracy. Traditionally, height systems are realized by applying the parametric adjustment and the least squares method. Any kind of correlation between the observations as well as the correlation of fixed heights is often neglected for pragmatical reasons. Also, given heights are usually assumed error-free when in reality they possess a certain level of accuracy. This paper presents an alternative realisation of the height reference system HVRS71 for the Republic of Croatia. Data scope is limited to the 1st and the 2nd order levelling networks. Alternative realisation is carried out by applying different adjustment concept where the correlations between the observations caused by the introduction of correlated given heights are considered. Also, this methodology enables the transfer of the accuracy of benchmarks' absolute heights from the higher-order levelling network to the accuracy of the benchmarks' absolute heights of lower-order levelling networks. Thus, an objective, logical and real distribution of the benchmarks' height accuracy along the scope of the height reference system is achieved. The results specify to what extent the change in the concept of the levelling network adjustment affects the quality of the height reference system.

KLJUČNE BESEDE

metoda najmanjših kvadratov, posredna izravnava, korelacija, višinski sistem, absolutna višina, natančnost višin

KEY WORDS

least squares method, parametric adjustment, correlation, height system, absolute height, height accuracy

1 INTRODUCTION

The reference frame, i.e., the realisation of the theoretical definition of the reference coordinate system, implies a firm materialization of geodetic points and the determination of their coordinates in the area of interest (Drewes, 2009). The quality of the reference frame establishment has a direct impact on the possibility of studying local and global changes on the Earth's surface, but it is also the basis for accurate and reliable geodetic products (Altamimi et al., 2001). Therefore, an objective insight into the level of quality of the reference frame is crucial. The realisation of the national height reference system is carried out by adjusting observed height differences in geometric levelling networks in relation to the selected height datum (Iliffe and Lott, 2008). In general, the primary goal of levelling network adjustment is to obtain unique values of the benchmarks' absolute heights and quality indicators of their height position. To this end, it is possible to apply different mathematical models, methods, and procedures. In other words, to choose different concepts of computational processing, which consequently give different results. Despite this, the basic purpose of adjustment, i.e., ensuring consistent and unique results, regardless of the chosen concept, will be and must always be fulfilled.

Usually, the adjustment is performed using the mathematical model of parametric adjustment and the least squares method. Thereat, all observations are assumed to be mutually uncorrelated and given heights error-free. The mentioned mathematical model consists of a functional part that contains a deterministic component of explicit functional connection between the observations and parameters of the model as well as a stochastic part defined by the weight matrix of the observations $\mathbf{P}$ (Kutterer, 1999; Teunissen, 2006a). The weight of an individual observation is a measure of its relative value in relation to other observations. It affects the residual values each observation receives within the adjustment (Ghilani and Wolf 2006). The weight matrix of the observations $\mathbf{P}$ is theoretically defined by the reference variance s_0^2 and the variances of individual observations s_i^2 combined in the variance matrix $\mathbf{V_{ll}}$ (Feil, 1989; Rožić 2007):

$$\mathbf{P} = s_0^2\,\mathbf{V}_{\mathbf{ll}}^{-1} = s_0^2 \begin{bmatrix} s_1^2 & 0 & \cdots & 0 \\ 0 & s_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_n^2 \end{bmatrix}^{-1} . \tag{1}$$

However, the weights of levelling observations can be defined differently. For instance, as reciprocal values of levelling line lengths or as reciprocal values of the number of levelling stations realised in a levelling line, in accordance with the laws of random errors propagation during measurement.

Following the algorithm of the parametric least squares adjustment, the calculation of adjusted heights $\bar{\mathbf{H}}$ is performed using the least squares solution $\mathbf{H}$ (Wolf, 1975):

$$\mathbf{H} = (\mathbf{A}^{t}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{t}\mathbf{P}\mathbf{l}, \tag{2}$$

where is: $\mathbf{A}$ – the design matrix,

$\mathbf{P}$ – the weight matrix of the observations,

$\mathbf{l}$ – the vector of reduced observations.

According to the expression (2), the values of adjusted heights are directly influenced by the shape and the configuration of the levelling network as well as the parameters of the introduced height datum and measurement accuracy. The quality assessment of adjusted heights expressed by the criterion of standard deviation s_{Hi} is calculated using the diagonal elements of the cofactor matrix of adjusted heights $\mathbf{Q_{HH}}$ i.e., cofactors of each benchmark height q_{HiHi} and the reference standard deviation s_0 in accordance with the following expressions:

$$\mathbf{Q_{HH}} = (\mathbf{A^tPA})^{-1}, \tag{3}$$

$$s_{Hi} = s_0\sqrt{q_{HiHi}}. \tag{4}$$

Expressions (2), (3) and (4) show that the formulation of the stochastic model in **P** has a direct influence on the results of the adjustment. Those are adjusted benchmark heights and their corresponding quality indicators.

However, the stochastic model, i.e. the weight matrix of the observations, can be structured in a more complex way than shown in expression (1) if mutual mathematical and/or physical correlation of observations is introduced into the stochastic model. Mathematical correlation occurs when quantities that are not directly measured, but are rather functions of some other direct observations, are introduced as observations into the functional model (Klak, 1982). Also, introducing some given heights of the model into the adjustment, which have been obtained as a result of some previously performed adjustment, induces their mathematical correlation. This is directly reflected in the structure of the weight matrix of the observations **P**. The elements on the main diagonal refer to observation variances, and elements outside the main diagonal to covariances (Wolf, 1975; Teunissen, 2006b). Namely, when uncorrelated observations are adjusted, the weight matrix of observations is diagonal and the covariances are equal to zero. In contrast, if the observations are correlated, the covariances are different from zero and the weight matrix **P** is not diagonal anymore. Moreover, if the stochastic model includes previous adjustments, the diagonal elements do not represent the original weights of the observations (Feil, 1990). In the context of a hierarchically structured levelling network adjustment, i.e., according to the orders of accuracy, given heights are often declared not only as uncorrelated, but also as non-error, although in reality there is a corresponding variance-covariance matrix obtained as a result of a previous adjustment. To simplify the computational process and in light of engineering pragmatism, aforementioned algorithm is widely used, and any kind of correlation is frequently neglected in geodetic practice.

There are other adjustment concepts that in different ways enable incorporation of information from previous adjustments which have been extensively studied (e.g. Mihailović, 1968; Wolf, 1975; Wolf, 1978; Hopcke, 1980; Mihailović, 1987; Feil, 1990; Miima, 1997; Ogonda, 2001). This paper presents the analysis of adjustment concept influence on height accuracy distribution using levelling networks in Croatia as an example.

2 HVRS71

The backbone of the Croatian height reference system - epoch 1971.5 (HVRS71) - is the Second High Accuracy Levelling Network (IINVT) with benchmarks' absolute heights determined in the system of the normal Earth's gravity field (normal-orthometric heights) in relation to the height reference surface which corresponds to the mean sea level determined from continuous tide gauge measurements over 18.6 years at five locations along the eastern coast of the Adriatic Sea (Rožić, 2019). The tide gauges are integrated into the levelling network by the connection to datum benchmarks, i.e., benchmarks stabilized in the immediate vicinity of the tide gauges. Their absolute heights determined in relation to the mean sea level have the function of height datum parameters. The initial establishment of the height reference frame is the result of the IINVT network adjustment. Generally, in the case of geometric levelling networks as an infrastructure of the national height reference coordinate system, there are levelling networks of different accuracies classified into orders (Mihailović, 1992). The levelling network IINVT is, in accordance with the *a priori* criterion of relative measurement accuracy, qualified as a 1st order network. Adjustment results encompass benchmarks' absolute heights as well as the corresponding quality indicators. This data also serves as support for the connection of all geometric levelling networks of lower orders. Moreover, the levelling networks of lower orders that are an integral part of the material basis of HVRS71 are the following: precise levelling (PN) – 2nd order, technical levelling of increased accuracy (TNPT) – 3rd order and technical levelling (TN) – 4th order.

The original adjustment of the levelling lines of the IINVT network (Klak et al. 1992) was carried out by applying the parametric least squares adjustment. The observations were treated as uncorrelated, and all parameters of the height datum were introduced into the adjustment as error-free. The weights of the observations were determined as reciprocal values of the levelling line lengths expressed in kilometres. The adjustment of all lower-order levelling networks was carried out hierarchically and gradually, by successively relying on the networks of higher orders. This is called the hierarchical principle. At the same time, benchmarks' heights of the higher-order network are declared error-free in the adjustment of the lower-order network, although they possess a certain level of accuracy. The adjustment was performed between nodal points, meaning that all observed height differences in the line between them were added together. The consequence is the availability of the absolute height accuracy indicator only for nodal benchmarks of levelling networks, and not for all benchmarks that define levelling sides. Determination of adjusted benchmarks' absolute heights within levelling lines was performed using the mathematical model of conditional adjustment and the least squares method. This model enables simpler calculation, but it is not possible to determine the absolute height accuracy of the benchmarks within each levelling line, using the standard algorithm.

The previously described concept and its effects can be illustrated using the example of the hierarchical relationship between the IINVT (1st order levelling network) and PN (2nd order levelling network) networks. The measurements of the IINVT network have been carried out in a relatively short period between 1970 and 1973. In contrast, measurements of the PN network have been executed for almost two decades, from 1945 to 1968. Fig. 1 shows the configuration scheme of these two levelling networks, i.e., the state levelling networks of the Republic of Croatia of the 1st and the 2nd order, the layout of the levelling loops and the location of the tide gauges which were used for height datum determination. In this figure, the positions of levelling lines of the IINVT network are geographically exact while the extension of the precision levelling network is only schematic.

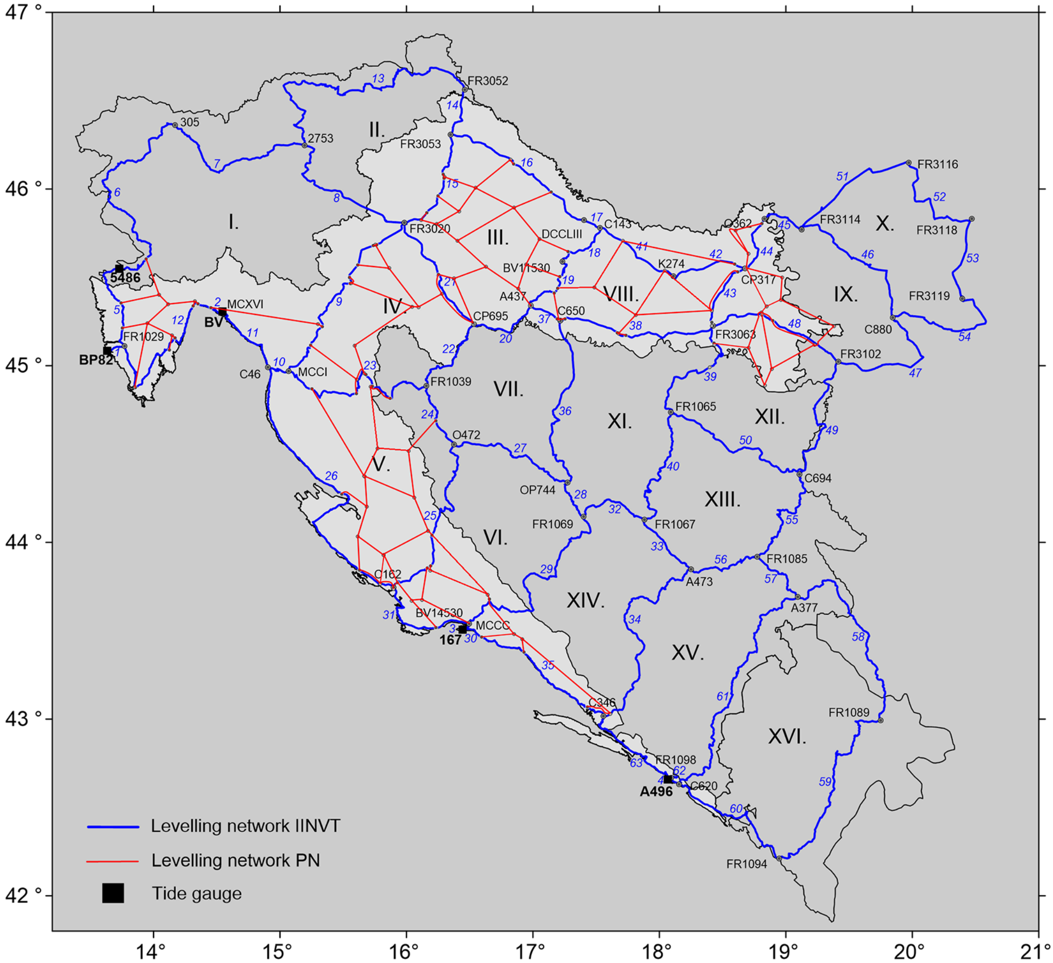

Figure 1: Configuration scheme of the IINVT and PN levelling networks on the Croatian territory

Firstly, in accordance with the adopted datum parameters and levelling survey data as well as the geometric configuration in Fig. 1, following the concept of the original realisation of HVRS71, the original adjustment of the IINVT levelling network was repeated. In addition to including all nodal benchmarks as unknown heights encompassed by the original adjustment, one configuration change was introduced. Specifically, the absolute heights of all benchmarks within the PN levelling lines that are connected to IINVT levelling lines were introduced as unknowns. In other words, a certain number of original IINVT levelling lines was structurally divided into several levelling line sections. This enables the determination of absolute height accuracy indicator for connection benchmarks as well. The mentioned configuration change does not affect the results of adjusted observations and heights relative to the original adjustment. However, it provides absolute height accuracy data for all connection benchmarks, which was not obtained in the original adjustment. A total of 126 measured height differences of levelling lines and sections of IINVT levelling lines were introduced into the adjustment as well as 106 unknown benchmarks' absolute heights, i.e., original nodal benchmarks of the IINVT network and connection benchmarks of the IINVT network for connecting the PN network. Adjustment was carried out by applying a mathematical model of parametric least squares adjustment, considering the observations as uncorrelated, height datum parameters as error-free and determining the weights of the observations as reciprocal values of levelling line lengths.

According to the accuracy indicators regarding the benchmarks' absolute heights, a distribution model of the benchmarks' absolute height accuracy is created (Fig. 2). Accuracy is expressed by the criterion of standard deviation of the benchmarks' absolute heights. The results are identical to the realisation contained in Rožić and Razumović, 2014. It should be noted that identical results were also achieved for all adjusted benchmarks' absolute heights.

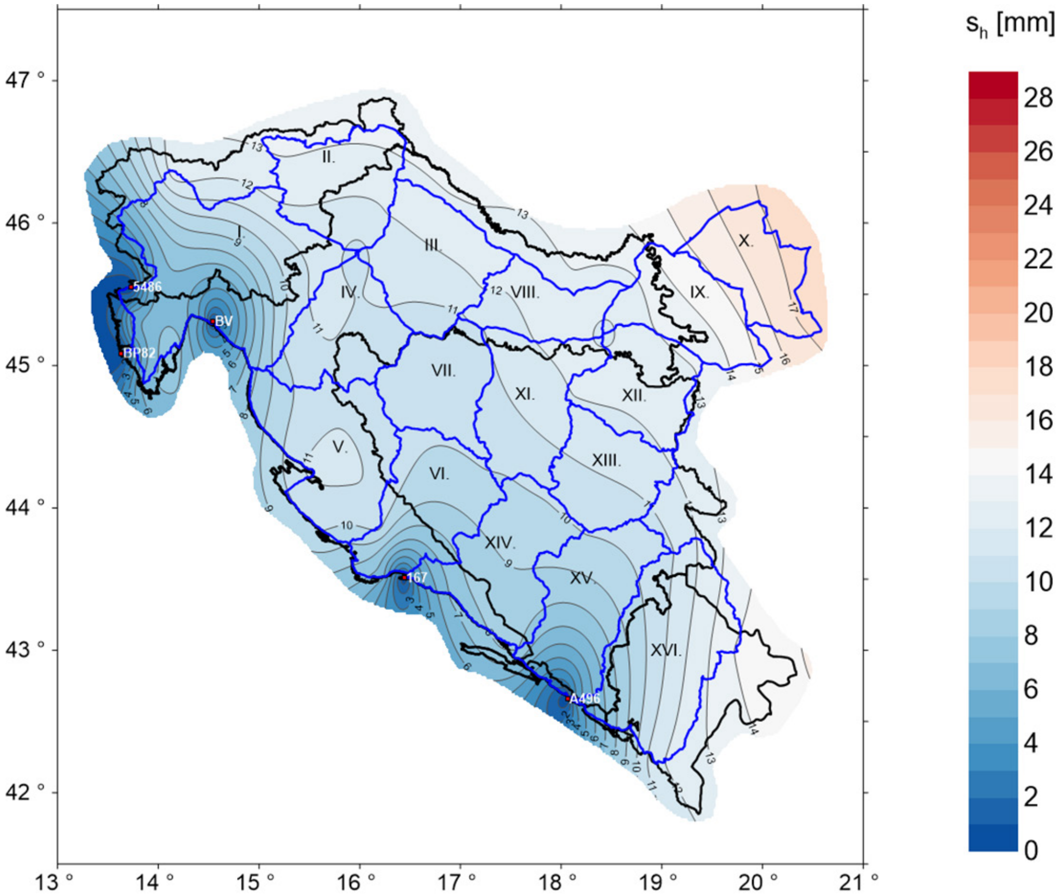

Figure 2: Height accuracy distribution of the benchmarks within the IINVT levelling network

The visualization of the benchmarks' absolute height accuracy in Fig. 2 indicates how the geometric configuration of the levelling network and fixing of datum parameters affect the distribution of the benchmarks' height accuracy along the area covered by HVRS71. Accuracy decreases with distance from the datum benchmarks stabilized at the tide gauge locations. Accordingly, maximum standard deviation values are recorded in the northeastern part of the network. This is a consequence of the five-parameter datum definition at the locations of the southwestern and southern edge of the IINVT network as well as of the propagation of levelling measurement errors with distance from the datum benchmarks. Isolines of the same accuracy show that faster and more dynamic accuracy decrease is present in the immediate vicinity of datum benchmarks. Moving away from them, the decrease is more moderate, and the trend is milder. Table 1 presents statistical indicators of calculated standard deviations of adjusted heights in the IINVT network. The minimum value of the standard deviation of the absolute height determination is observed for the nodal benchmark MCCC which equals 0.49 mm. This benchmark is located in the immediate vicinity of the datum benchmark 167 at the tide gauge in Split (Fig. 2). The maximum value of the standard deviation reaches the value of 17.65 mm, and it is determined for benchmark FR3118, which is in the northeast of the IINVT levelling network, also outside the borders of the Republic of Croatia.

Table 1: Statistical indicators of calculated standard deviations of adjusted heights in the IINVT network

	Standard deviation of the adjusted heights
Minimum	0.49 mm
Maximum	17.65 mm
Mean	10.43 mm
Range	17.15 mm

Secondly, and in accordance with the hierarchical principle, the PN levelling network has been adjusted also using the mathematical model of parametric least squares adjustment, treating the observations as uncorrelated. The original adjustment of the PN levelling network was performed individually for each levelling figure of the IINVT out as a part of the original realisation of the HVRS71. In contrast to that, this adjustment has been carried out integrally for the whole levelling network. The PN network relies on the connection benchmarks with previously determined heights in the IINVT network adjustment. Thereat, 144 levelling lines were included, as well as 72 unknown absolute heights of nodal benchmarks of the PN network. The weights of observations are also determined as reciprocal values of the levelling line lengths. In accordance with the original concept of the HVRS71 realisation, all connection benchmarks are introduced into the adjustment as error-free.

Fig. 3 shows the distribution model of the absolute height accuracy of the realised height reference system generated based on the adjustment of the hierarchically connected levelling networks of the 1st and the 2nd order. This model reflects the adopted concept of computational processing, in which there is no transfer of the absolute height accuracy of the benchmarks from the higher-order network to the lower-order network. This is because the connection benchmarks contained in the higher-order network are fixed and declared as error-free in the adjustment process, although they possess a certain level of accuracy (as seen in Fig. 2). It is possible to declare these benchmarks as error-free since they belong to a higher-order network, and it is known that hierarchically superior order network has higher level of measurement accuracy in relation to the subordinate order network.

In comparison to Fig. 2 where the distribution of the benchmarks' height accuracy has been generated only on the basis of the IINVT network adjustment, in Fig. 3 there is a clear and general change in the benchmarks' height accuracy distribution. A rather illogical accuracy increase at certain locations within the area covered by the network is observed. Table 2 presents statistical indicators of calculated standard deviations of adjusted heights in the PN network obtained from the adjustment using the hierarchical

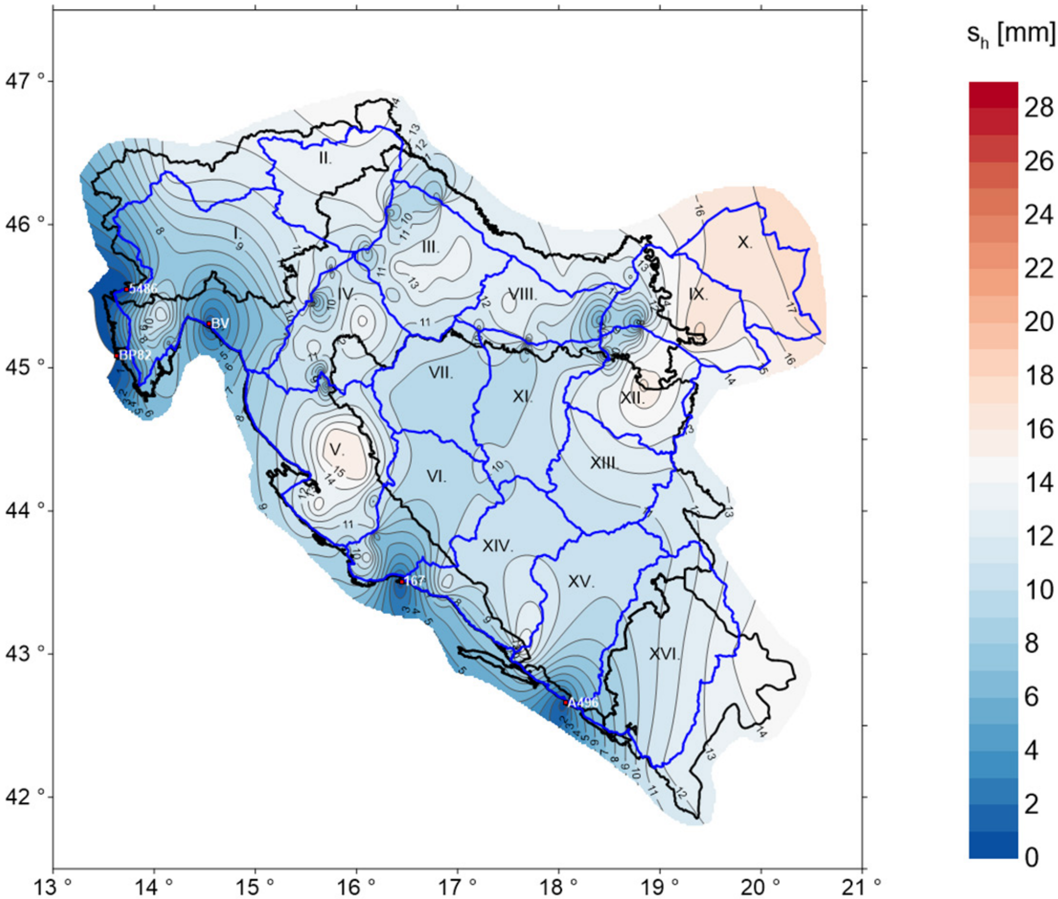

Figure 3: Height accuracy distribution of the benchmarks within the IINVT and PN levelling networks obtained from the adjustment using the hierarchical principle

principle. The benchmark with the smallest value of standard deviation of 0.12 mm is BV10877, and it is located on the eastern border of the Republic of Croatia, which belongs to the levelling figure IX. One can clearly notice the illogicality that, based on the comparison between the models in Fig. 2 and Fig. 3, some benchmarks that belong to the 2nd order network achieve a higher level of height accuracy than the accuracy determined by the adjustment of the 1st order network. For instance, when adjusting the levelling network IINVT, the minimum value of the absolute height standard deviation of the benchmark MCCC equals 0.49 mm (Table 1). This is quite logical because this benchmark is near the datum benchmark at the tide gauge in Split. Whereas when adjusting the levelling network PN, the minimum value of the absolute height standard deviation equals 0.12 mm at benchmark BV10877, which is on the northeastern edge of the height reference system within the territory of the Republic of Croatia, far away from the tide gauges (Table 2). These results are not consistent with the results obtained in the previous adjustment of the IINVT network as well as the conclusions that came out from it. Levelling measurement errors should increase with distance from the datum benchmarks which is not the case here. In addition, benchmarks belonging to the 2^{nd} order network achieve higher accuracy level than the benchmarks in the 1^{st} order network. This illogical result is the consequence of the adjustment concept and method applied. On one hand any kind of correlation between PN and IINVT benchmarks was neglected. On the other hand, accuracy indicators of benchmark heights in IINVT network were declared as error-free.

Table 2: Statistical indicators of calculated standard deviations of adjusted heights in the PN network obtained from the adjustment using the hierarchical principle

	Standard deviation of the adjusted heights
Minimum	0.12 mm
Maximum	17.54 mm
Mean	9.62 mm
Range	17.42 mm

3 ALTERNATIVE REALISATION OF THE HVRS71

To overcome the illogicality aforementioned, it is possible to apply different adjustment concepts. A common least square adjustment of IINVT and PN networks could be the easiest solution. In that case, special attention must be paid to stochastic model definition. Since these two networks are heterogenous and belong to different orders of accuracy (Rožić, 2019), their weights have to be defined accordingly. Additional problem is to find the appropriate method to estimate the weight relation between these observations (Ogonda, 2001). However, the limitations of a common adjustment are the change in benchmarks' heights with every single observation added to the network because the whole adjustment must be repeated (Mihailović, 1987; Ogonda, 2001). The magnitude of value changes in benchmarks' heights depends on the number of new observations. The result is loss of meaning of national reference system (Ogonda, 2001). In addition, this is not in accordance with the basic geodetic guiding principle "from whole to part".

An alternative method that could be applied is to account for the accuracy indicators of given heights obtained as a result of previous adjustment (IINVT benchmarks) but without changing their heights. In this paper, the alternative realisation of the HVRS71 is carried out using the mathematical model of parametric least squares adjustment of correlated measurements (Hopcke, 1980; Feil, 1990). This algorithm is almost completely consistent with previously described least square parametric adjustment that is usually used in geodetic practice. The only and key difference is contained in the stochastic part of the model and its genesis. We will illustrate this model on 1D example with just one observed height difference. Parametric equation connecting the observation, and the unknown benchmark height is equal to the expression:

$$\Delta h_1 + v_1 = \bar{H} - H_1 \qquad (5)$$

where Δh_1 is the observed height difference, v_1 is the observation residual (correction to be applied to observation), $\bar{H}$ is the adjusted benchmark height which is unknown and has to be determined in the adjustment, and H_1 is the given benchmark height that is already known and obtained from a previous adjustment.

In this case, the parametric equation is already linear so there is no need for linearization using the Taylor's series expansion. Considering that adjusted benchmark height $\overline{\mathrm{H}}$ is the sum of approximate benchmark height H_0 and least squares solution defined in the expression (2), parametric equation can be rewritten in a following way:

$$v_l = H + (H_0 - H_l - \Delta h_l). \tag{6}$$

The last part of this equation in the brackets represents the reduced observation $-l_l$. Likewise, parametric equations and reduced observations can be formulated for all observations included in the adjustment as it follows:

$$-l_l = H_{oj} - H_k - \Delta h_i. \tag{7}$$

where i = 1, ..., n (n is the number of observations), j = 1, ..., u (u is the number of unknown benchmark heights that have to be determined in the adjustment) and k = 1, ..., r (r is the number of given benchmark heights obtained from the previous adjustment). All these reduced observations form a vector of reduced observations $-\mathbf{l}$. If we apply the law of cofactor propagation to all reduced observations from the expression (7) the result is (Feil, 1990):

$$\mathbf{Q_{ll}} = \mathbf{Q_{LL}} + \mathbf{Q_{ff}}. \tag{8}$$

$\mathbf{Q_{LL}}$ is the diagonal cofactor matrix of the observations, calculated as inverse of the weight matrix of the observations according to the expression (1). In our case, those are the observations of the PN levelling network. To calculate the matrix $\mathbf{Q_{ff}}$ it is necessary to formulate the matrix **F** which represents the coefficient matrix containing partial derivatives of the functions of reduced observations with respect to the variables from the previous adjustment and its dimension is n×r. Here, the partial derivatives are either 0 or 1. The cofactor matrix of adjusted benchmark heights $\mathbf{Q_{HH}}$, as specified in expression (3), is known as a result of the previous adjustment and its dimension is r×r. In this example, this is the cofactor matrix of adjusted benchmark heights within IINVT levelling network. The matrix $\mathbf{Q_{ff}}$ can be calculated as it follows:

$$\mathbf{Q_{ff}} = \mathbf{F Q_{HH} F^t}. \tag{9}$$

Finally, the weight matrix of correlated observations $\mathbf{P_{ll}}$ is defined as:

$$\mathbf{P_{ll}} = \mathbf{Q_{ll}}^{-1}. \tag{10}$$

This concept allows taking into account the calculated height accuracy indicators of 1st order benchmarks and transfer them to the 2nd order benchmarks. At the same time, heights of the IINVT connection benchmarks are fixed and do not change. Previously presented methodology was used for stochastic model definition and the least square adjustment was performed. A total of 144 measured height differences of the PN levelling lines with known levelling line lengths and 72 unknown absolute heights of nodal benchmarks of the PN network were introduced into the adjustment. In addition to that, 64 connection benchmarks of the levelling network IINVT on which PN observations rely as well as their cofactor matrix (QHH) were also included into the adjustment. Based on the performed adjustment, a distribution model of the benchmarks' absolute height accuracy is presented in Fig. 4.

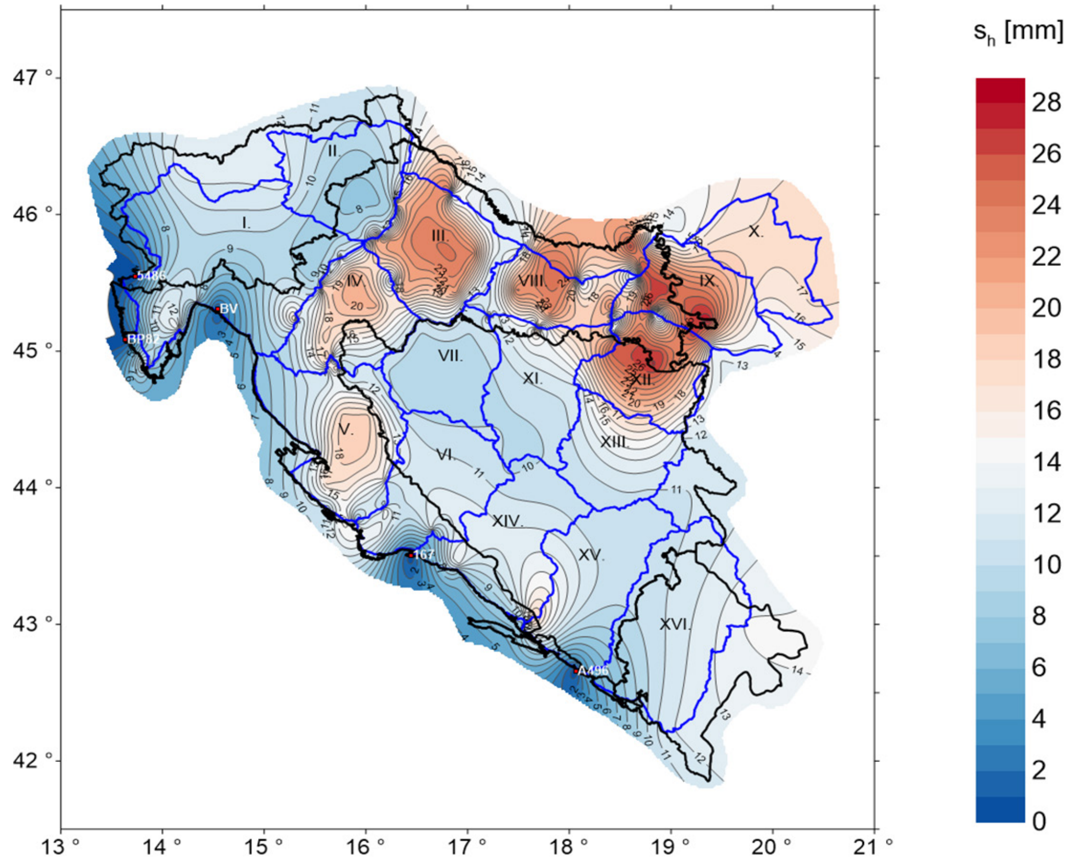

Figure 4: Height accuracy distribution of the benchmarks of the IINVT and PN levelling networks obtained from the parametric least squares adjustment of correlated measurements

Comparing the Fig. 3 and Fig. 4, it is quite clear that in both cases, i.e., in both applied concepts of network adjustment, a significantly different level and distribution of the benchmarks' absolute height accuracy along the scope of the height reference system is achieved. Two interesting and, at the same time, completely logical phenomena can be noted. First, standard deviations of PN benchmarks' absolute heights obtained from the parametric least squares adjustment of correlated measurements (Fig. 4) logically and consistently for all benchmarks take on higher values compared to the results from the adjustment where the correlation was neglected and the transfer of accuracy not present (see Fig. 3). Second, there is a noticeable and logical occurrence of a decrease in the benchmarks' height accuracy from the edges to the centres of individual levelling figures of the IINVT network, which is a consequence of the increase in levelling measurement errors. Table 3 presents statistical indicators of calculated standard deviations of adjusted heights in the PN network obtained from the parametric least squares adjustment of correlated measurements. On one hand, the minimum value of standard deviation of 11.29 mm refers to the PN benchmark 2797 in the immediate vicinity of the datum benchmark BV connected to the tide gauge in Bakar. On the other hand, the maximum value of standard deviation of 27.47 mm is observed by the benchmark MCCXI in the extreme northeast of the height reference system. This result is in accordance with the expected propagation of levelling measurement errors with distance from the datum benchmarks. Also, in this case benchmarks that belong to the 1st order network achieve higher level of accuracy than the 2nd order benchmarks. Extreme values of height standard deviation in these two adjustment concepts differ from each other by approximately 1 cm (Table 2, Table 3).

Table 3: Statistical indicators of calculated standard deviations of adjusted heights in the PN network obtained from the parametric least squares adjustment of correlated measurements

	Standard deviation of the adjusted heights
Minimum	11.29 mm
Maximum	27.47 mm
Mean	20.08 mm
Range	16.19 mm

In view of the above, it is undoubted that both mathematical models of adjustment that were applied, each lead to unambiguous and consistent values of the benchmarks' absolute heights and their accuracy. However, considering the genesis of the stochastic model, it is also undoubted that different concepts of computational processing of observations unambiguously affect the amounts and distribution of the benchmarks' height accuracy. This is a consequence of introducing or not introducing the correlation of observations and especially transferring the height accuracy from the benchmarks of the higher-order levelling network to the benchmarks of the lower-order networks.

4 CONCLUSION

In case of computational processing of hierarchically structured levelling networks, it is possible to apply different concepts and procedures for the realization of height reference system. All variants have in common that each carries out the purpose of adjustment, i.e., unambiguous values of the absolute heights of benchmarks and their accuracy. Although the results between individual variants are clearly different. Parametric least squares adjustment is traditionally used in the realisation of height reference systems based on hierarchically structured levelling networks. The disadvantage of this traditional approach is the nullification of the connection benchmarks' accuracy for connecting lower-order networks and neglecting the mathematical correlation between the absolute heights of connection benchmarks. The consequence is the impossibility of a logical and objective transfer of height accuracy from the benchmarks of the higher-order levelling network to the benchmarks of the lower-order networks.

In this paper, an alternative realisation of the HVRS71 is carried out by applying the parametric least squares adjustment of correlated measurements. This method accounts for the correlation between measurements and allows the transfer of height accuracy but keeping the heights of higher-order network benchmarks unchanged. Comparison of results obtained from the original and alternative realisation indicates significant differences in terms of value and trend of the distribution of the benchmarks' height accuracy. At the same time, the appearance of a decrease in the benchmarks' height accuracy within individual figures of the 1st order network with distance from the edges of the figures towards their centre, is quite evident.

The application of the alternative concept of the HVRS71 realisation proves to be more objective, realistic, and logical compared to the original realisation. In the specific case, and according to the available data, only the 1st and the 2nd order levelling networks are included in the adjustment, because even without the consistent inclusion of the 3rd and 4th order networks, the essence of the obtained results has been pointed out.

Literature and references:

Altamimi, Z., Angermann, D., Argus, D., Blewitt, G., Boucher, C., Chao, B., Drewes, H., Eanes, R., Feissel, M., Ferland, R., Herring, T., Holt, B., Johannson, J., Larson, K., Ma, C., Manning, J, Meertens, C., Nothnangle, A., Pavlis, E., Petit, G., Ray, J., Ries, J., Scherneck, H.G., Sillard, P. and Watkins, M. (2001). The terrestrial reference frame and the dynamic Earth. Eos, Transactions American Geophysical Union, 82 (25), 273-279. DOI: https://doi.org/10.1029/EO082i025p00273-01

Drewes, H. (2009). Reference Systems, Reference Frames, and the Geodetic Datum. In: Sideris, M.G. (eds) Observing our Changing Earth. International Association of Geodesy Symposia, Springer, Berlin, Heidelberg, vol 133, 3-9. DOI: https://doi.org/10.1007/978-3-540-85426-5_1

Feil, L. (1989). Teorija pogrešaka i račun izjednačenja I dio. [Error Theory and Adjustment Computations 1st part]. [In Croatian]. Faculty of Geodesy, University of Zagreb, Zagreb.

Feil, L. (1990). Teorija pogrešaka i račun izjednačenja II dio. [Error Theory and Adjustment Computations 2nd part]. [In Croatian]. Faculty of Geodesy, University of Zagreb, Zagreb.

Ghilani, D. C. and Wolf, P. R. (2006). Adjustment Computations. Spatial Data Analysis, John Wilwy & Sons. Inc, Hoboken, New Jersey, SUA.

Hopcke, W. (1980). Fehlerlehre und Ausgleichungsrechnung. [Error Theory and Adjustment Computations]. [In German]. Berlin and New York.

Iliffe, J. C. and Lott, R. (2008). Datums and Map Projections for Remote Sensing. GIS and Surveying, 2nd edition, Dunbeath: Whittles Publishing.

Klak, S. (1982). Teorija pogrešaka i račun izjednačenja. [Error Theory and Adjustment Computations]. [In Croatian]. Liber, Zagreb.

Klak, S., Feil, L. and Rožić, N. (1992). Studija o sređivanju geometrijskog nivelmana na području Republike Hrvatske. [A Study on the Geometric Levelling Arrangement in the Territory of the Republic of Croatia]. [In Croatian]. Faculty of Geodesy, University of Zagreb, Zagreb.

Kutterer, H. (1999). On the Sensitivity of the Results of Least-Squares Adjustments Concerning the Stochastic Model. Journal of Geodesy 73, 350–361. DOI: https://doi.org/10.1007/s001900050253

Mihailović, K. (1968). Neki novi aspekti o izravnanju geodetskih mreža. [Some New Aspects of Geodetic Network Adjustment]. [In Serbian]. Geodetski list, 22(4–6), 69-72.

Mihailović, K. (1987). Geodezija II, prvi deo. [Geodesy II, 2nd part]. [In Serbian]. Naučna knjiga. Beograd.

Mihailović, K. (1992). Geodezija: izravnavanje geodetskih mreža. [Geodesy: Adjustment of Geodetic Networks]. [In Serbian]. Naučna knjiga. Beograd.

Miima, J. B. (1997). An Evaluation of the Static, Dynamic, and Static-dynamic Geodetic, Densification Models on a Part of the Kenyan Geodetic Network. Doctoral dissertation, University of Nairobi.

Ogonda, G. O. (2001). Densification of geodetic control Networks under reproducing parametric and stochastic fiducial constraints. Doctoral dissertation, University of Nairobi.

Rožić, N. (2007). Računska obrada geodetskih mjerenja. [Computational Processing of Geodetic Measurements]. [In Croatian]. Faculty of Geodesy, University of Zagreb, Zagreb.

Rožić, N. and Razumović, I. (2014). High Accuracy Levelling Network Design Issue on the Territory of the Republic of Croatia. In:14th International Multidisciplinary Scientific GeoConference SGEM 2014, Albena, Bulgaria. Conference proceedings, Section Informatics, Geoinformatics and Remote Sensing, Volume II. 307-314.

Rožić, N. (2019). Hrvatski visinski referentni sustav. [Croatian Height Reference System]. [In Croatian]. University of Zagreb, Faculty of Geodesy. Chair of Analysis and Processing Geodetic Measurements, Zagreb.

Teunissen, P. J. (2006a). Adjustment Theory. VSSD. Delft.

Teunissen, P. J. (2006b). Network Quality Control. VSSD. Delft

Wolf, H. (1975.). Ausgleichungsrechnung. Formel zu praktischen Anwendung. [Adjustment Computations. Formula for Practical Application]. [In German]. Ferd. Dümmlers Verlag. Bonn.

Wolf, H. (1978). Ausgleichungsrechnung II: Aufgaben und Beispiele zur praktischen Anwendung. [Adjustment Computations II: Tasks and Examples for Practical Application]. [In German]. Ferd. Dümmlers Verlag. Bonn

Bukač B., Razumović I., Rožić N. (2024). The Impact of the Adjustment Concept on the Benchmarks' Height Accuracy.
Geodetski vestnik, 68 (4), 419-431.
DOI: https://doi.org/geodetski-vestnik.2024.04.419-431

Blaženka Bukač, mag. ing. geod. et geoinf.
University of Zagreb, Faculty of Geodesy
Kačićeva 26, HR-10000 Zagreb, Croatia
e-mail: blazenka.bukac@geof.unizg.hr

Prof. Nevio Rožić Ph.D.
University of Zagreb, Faculty of Geodesy
Kačićeva 26, HR-10000 Zagreb, Croatia
e-mail: nevio.rozic@geof.unizg.hr

Assoc. Prof. Ivan Razumović Ph.D.
University of Zagreb, Faculty of Geodesy
Kačićeva 26, HR-10000 Zagreb, Croatia
e-mail: ivan.razumovic@geof.unizg.hr