OCENA RAZLIK MED VIŠINSKIM DATUMOM JUŽNE AFRIKE IN DATUMOM MEDNARODNEGA VIŠINSKEGA REFERENČNEGA SISTEMA

ESTIMATION OF VERTICAL DATUM OFFSET FOR THE SOUTH AFRICAN VERTICAL DATUM, IN RELATION TO THE INTERNATIONAL HEIGHT REFERENCE SYSTEM

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IZVLEČEK

Razlika in vrednost težnostnega potenciala v Južni Afriki se ocenjujeta na štirih temeljnih referenčnih točkah, in sicer s primerjavo z mednarodnim višinskim referenčnim sistemom IHRS. Predvideva se, da obstajajo razlike med višinskim datumom Južne Afrike (W_P) in globalnim višinskim datumom (W_0). V raziskavi je bil uporabljen pristop rešitve problema geodetskega robnega pogoja za eno točko (angl. geodetic boundary value problem – GBVP*), kjer smo z Brunsovo enačbo ocenili vrednost anomalije višine po teoriji Molodenskega iz tako imenovanega motečega potenciala (T_P). Na obravnavanih mareografih v Južni Afriki je težnostni potencial odstopal od globalnih referenčnih vrednostih za naslednje vrednosti: Cape Town 0,589 m^2s^{-2}, Port Elizabeth-1,993 m^2s^{-2}, East London -2,593 m^2s^{-2}, Durban2,154 m^2s^{-2}. Odmik višinskega datuma na obravnavanih točkah glede na mednarodni višinski referenčni sistem je tako 6,013 cenitmetra v Cape Townu, -20,347 cenitmetra v Port Elisabethu, -26,478 cenitmetra v East Londonu in 21,996 cenitmetra v Durbanu. Ugotovljene razlike se lahko uporabljajo za uskladitev višinskega datuma Južne Afrike z mednarodnim višinskim referenčnim sistemom.*

ABSTRACT

The vertical offset and the geopotential value over South Africa is estimated on the four fundamental benchmarks in relation to the international height reference system (IHRS). It is estimated to obtain discrepancies between the South African local vertical datum (W_P) and the global vertical datum (W_0). A single-point-based geodetic boundary value problem (GBVP) approach was used following Molodensky theory for estimating the height anomalies from the disturbing potential (T_P) using Bruns's formula. The gravity potential at each tide gauge benchmark (TGBM) in South Africa deviates from the potential of the global reference surface by 0.589,-1.993,-2.593 and 2.154 m^2s^{-2} for Cape Town, Port Elizabeth, East London and Durban, respectively. The corresponding vertical datum offsets between the international height reference system and the four fundamental benchmarks over South Africa are 6.013, -20.347, -26.478, and 21.996 cm for Cape Town, Port Elizabeth, East London and Durban, respectively. These offsets can be used for the unification of the South African vertical datum at the four tide gauge benchmarks in a manner that is consistent to the international height reference system.

KLJUČNE BESEDE

geoid, kvazigeoid, težnostni potencial, višinski datum, moteči potencial, anomalija višine

KEY WORDS

Geoid, quasigeoid, geopotential, vertical datum, disturbing potential, height anomaly

1 INTRODUCTION

The resolution for the development of an international height reference system (IHRS) was released by the International Association of Geodesy (IAG) in July 2015 (IAG, 2015). The IHRS was developed to provide a global vertical reference system of high precision. This will provide support in monitoring global changes, geohazards, and prediction of several Earth's science phenomena. The IHRS is defined by an equipotential surface of the Earth's gravity field realised by a conventional value, W_0 = 62,636,853.4 m^2s^{-2} (Burša et al., 2001; 2004; Sánchez et al., 2016; Sánchez and Sideris, 2017). However, a number of recent researches have shown that this value may have increased by 1 – 2 m^2s^{-2} (Rülke et al., 2013; Albarici et al., 2019). The value of W_0 in practice depends on the realisation of the vertical datum (Amjadiparvar et al., 2013).

The South African land levelling datum (LLD) has been providing the reference frame for a variety of practical applications, such as the construction of roads, the development of infrastructures and a variety of developmental activities in the country. The South African LLD was realized over a century ago, based on mean sea level (MSL) observations from four tide gauge stations (situated in Cape Town, Port Elizabeth, East London and Durban). It was connected to the national benchmark network by primary levelling networks, which were adjusted independently. In addition, heights measured above LLD are classified as spheroidal orthometric height system. This height system provides a poor approximation of the true orthometric height system. However, it is estimated to be more closer to the normal height system (Merry, 1985).

In this height system, the spheroidal orthometric correction was applied to all the height differences from the primary levelling networks, computed from normal gravity. However, the orthometric correction was computed for only four levelling loops around Cape Town, meaning that the actual gravity measurements were taken for only those loops (Merry, 1977, 1985; Wonnacott & Merry, 2011). The spheropotential number is used in this height system instead of the geopotential number which is derived from the normal gravity (Odumosu et al., 2015).

The South African vertical datum suffer from a number of problems such as; numerous errors from the levelling networks and tide gauge sea level measurements, instability due to high MSL variability, and it was established from inconsistent levelling networks, just to mention a few. In addition, it has been established by Merry (2003) that the South African LLD is 15–20 cm below the mean sea level. Therefore, in order for South Africa to meet the standards of the global vertical datum, the South African vertical datum must be unified and also be defined by a gravity potential value. This will provide South Africa with a modernised vertical datum.

To achieve this, the South African vertical datum should be defined by means of a geoid model; this approach will solve some of the problems associated with the LLD. The main focus of this study is to estimate the vertical datum offset for the South African vertical datum, at the four fundamental benchmarks, in relation to the IHRS. The national primary levelling networks can be adjusted using geopotential difference instead of height differences. This is conducted by studying the relationship between gravity potential and height in the vertical datum definition and realisation. The growing need for a global reference surface requires a unification of all existing vertical datums around the world, which is a scientific problem of high practical significant (Sánchez et al., 2018).

Unification of height systems requires the determination of the transformation parameters or datum offsets between existing vertical datums, each of which is defined with a fundamental surface of zero elevation. Vertical datum offset is an existing discrepancy between datums; it can be estimated from GPS/levelling data of benchmarks on land, GPS/levelling data of tide gauge stations, Global Geopotential Models (GGM) and a precise geoid model (Singh, 2018). Presently, it is common practice for a vertical reference surface to be defined by a gravimetric geoid model.

Traditionally, national and regional height datums were defined with respect to a selected network of tide gauge stations; and height networks were established by terrestrial techniques such as spirit levelling. Height differences (*dH*) measured during levelling are scaled by gravity (*g*) to determine the difference in gravity potential (*dW*, also known as a change in gravity potential), this relationship can be expressed as follows (Heiskanen & Moritz, 1967):

$$dW = g \times dH \tag{1}$$

The difference in gravity potential is known as geopotential number (C_P), in this study, it is defined as the difference between the constant gravity potential at the global geoid (W_0) and the gravity potential at the point *P* on the local geoid (W_P) it can be expressed as follows (Heiskanen & Moritz, 1967):

$$C_P = W_0 - W_P = -\int_0^P dW = -\int_0^P g dH, \tag{2}$$

If W_P and W_0 could be measured and defined respectively, an ideal height system could be determined. The negative sign in the equation above indicates that an increase in height invokes a decrease in gravity potential. It should also be noted that over a short or in regions of low gravitational variation, the geopotential number will be insignificant (Heiskanen and Moritz, 1967).

2 VERTICAL DATUM

The geoid is commonly known as the surface of equal geopotential; the numerical value of the geopotential of the global geoid has been determined from analysis of satellite tracking data, GPS/levelling data and satellite altimetry measurements. A vertical reference frame is a reference network consisting of a set of physical reference points, whose vertical coordinates refer to the reference system measured within that frame. Meanwhile, the vertical datum is defined as the zero-level surface (Sánchez and Sideris, 2017; Zhang et al., 2020). A local vertical datum is usually defined by a fundamental benchmark/s or point/s of origin, related to the mean sea level at tide gauge station(s).

Over the years, many different types of vertical datums have been used. To name a few examples of datums and their related height system, heights derived from GPS observations are referred to as ellipsoidal heights (*h*) have as a datum the ellipsoidal surface, the orthometric height (H^0) derived from traditional spirit levelling measured above a geoidal surface, the normal height (H^N) measured above a quasigeoid surface, and the spheroidal orthometric height (H^{LLD}) measured above the land levelling datum (LLD), as depicted in Figure 1.

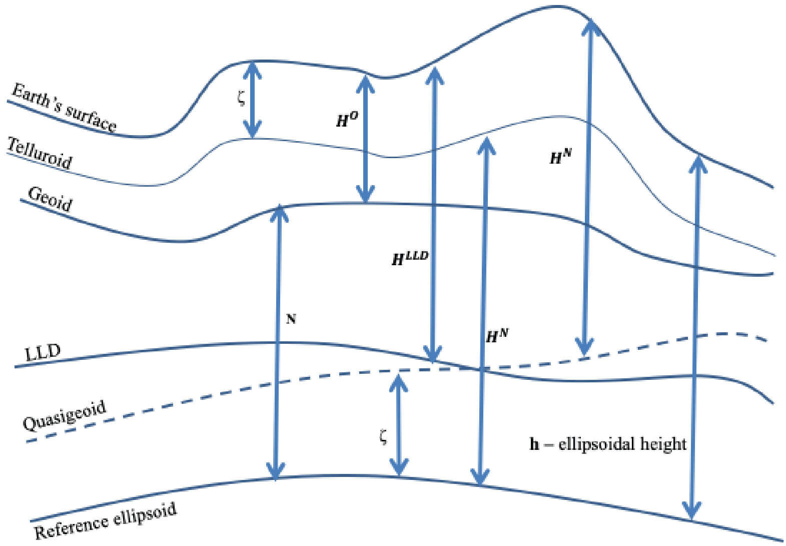

Figure 1: Relationship between common height systems.

Ellipsoidal heights are geometric quantities with no physical meaning; it is only practical if the information of the geoid undulation (N) is available and also it is measured solely from space-based instruments. However, orthometric and normal heights are the most commonly used height systems, in which height differences can be represented in potential differences, as defined in section 1. These height systems can be expressed as follows (Heiskanen & Moritz, 1967):

$$H^O = \frac{C_P}{\bar{g}} = \frac{W_0 - W_P}{\bar{g}}; \; H^N = \frac{C_P}{\bar{\gamma}} = \frac{W_0 - W_P}{\bar{\gamma}} \tag{3}$$

where $\bar{g}$ and $\bar{\gamma}$ are mean actual and normal gravity along actual and normal plumb-lines through point P (on the Earth surface), respectively. In general, national vertical datums are defined by selecting fundamental Benchmark/s at coastal tide gauge stations and setting $N = 0$, $W_0 = W_P$, and then they are connected to the national levelling network.

In this study, vertical datum offsets are estimated using a single-point-based Geodetic Boundary Value Problem (GBVP) approach following Molodensky's theory for estimating the height anomalies from the disturbing potential using Bruns's formula. The vertical datum offset is only estimated at the four fundamental benchmarks to be able to unify the South African vertical datum to the global vertical datum.

3 THEORETICAL BACKGROUND

The relationship between the gravity potential (W) and the corresponding normal potential (U) of the reference ellipsoid can be determined from estimating the disturbing potential (this can be expressed as follows (Heiskanen and Moritz, 1967):

$$W_P = U_P + T_P \tag{4}$$

The normal potential at a point on the Earth surface is determined as follows (Heiskanen & Moritz, 1967):

$$U_P = U_0 + \frac{\partial U_0}{\partial h} h_P, \tag{5}$$

Where h_p represents an ellipsoidal height at the point P, U_0 is the normal gravity potential obtained directly from the World Geodetic System 1984 (WGS84) reference ellipsoid and $\frac{\partial U_0}{\partial h}$ is the gradient of normal gravity potential. In this study, a single-point-based GBVP approach is employed to determine the vertical datum offset for height system unification. This is done by following Molodensky theory for estimating the height anomalies from the disturbing potential using Bruns's formula.

The disturbing potential at the point P is computed from the spherical harmonic coefficients of the latest Gravity field and steady-state Ocean Circulation Explorer (GOCE) based GGM (to degree 300) TIM6 (Zingerle et al., 2019). According to Odera (2019), the GOCE-based GGM, especially the timewise solution (TIM) has the best agreement with the latest gravimetric quasigeoid model over South Africa. It was integrated with the residual gravity anomalies ($\Delta g - \Delta g_{GGM}$) using Stokes's integral while residual terrain model (RTM) was used to cater for the contribution of short wavelength component. This was done by evaluating Stokes's integral of the gravity anomalies combined with the Molodensky G_1 term. The solution to the GBVP at point P in terms of the disturbing potential is expressed as follows (Torge & Muller, 2012), this is usually referred to as the remove-compute-restore procedure,

$$T_P = T_{GGM} + \frac{R}{4\pi} \iint_{\sigma} (\Delta g - \Delta g_{GGM} + G_1) \times S(\psi)\, d\sigma + T_{RTM} \tag{6}$$

Where, R – a mean radius of the Earth, Δg – free-air gravity anomalies, Δg_{GGM} – gravity anomalies generated by the GGM, ψ – geocentric angle/ spherical distance, $d\sigma$ – an infinitesimal surface element of the unit sphere σ (corresponding to ellipsoidal coordinates), $S(\psi)$ Stokes's function, T_{GGM} – long-wavelength component of the disturbing potential. The Stokes's Kernel function can be computed as expressed by equation (7) (Heiskanen & Moritz, 1967), the Stokes' integral in equation (6) was evaluated using the technique described in detail by Yun (1999) (see section 3, eq. 6), a brief elaboration of the technique for computer programming was given by Bracewell (1978).

$$S(\psi) = \frac{1}{\sin(\frac{\psi}{2})} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi - 3\cos\psi . \ln\left[\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right]. \tag{7}$$

The contribution of the RTM to the disturbing potential (T_{RTM}) was evaluated as follows (Forsberg, 1985):

$$T_{RTM} = -\pi G \rho H_P^2 - \frac{G\rho R^2}{6} \iint_{\sigma} \frac{(H^3 - H_P^3)}{\ell^3} d\sigma \tag{8}$$

Where H *and* H_P are the heights of roving point and computation point, respectively, G – is the Newtons gravitational constant, ρ – is the topographic mass density distribution, $\rho = 2670\ kg.m^{-3}$, and l – is the planar distance between the computational point and the roving points. The residual gravity anomalies are in principle converted into residual disturbing potential, using 2D Fourier transform with a spherical approximation of the RTM terrain correction integration (Yun, 1999). Moreover, a digital elevation model (DEM) from the Shuttle Radar Topography Mission (SRTM) at 90 m spacing was used to evaluate the Molodensky G_1 term (see equation (6)), this can be expressed as follow (McCubbine et al., 2018):

$$G_1 = \frac{\Delta\varphi\Delta\lambda}{2\pi}\left[\left(H.\Delta g\right)\times\frac{1}{\ell^3} - \mathrm{H_P}\left(\Delta g\times\frac{1}{\ell^3}\right)\right] \tag{9}$$

Where $\Delta\varphi$ and $\Delta\lambda$ are the differences in latitude and longitude, respectively. The G_1 term contribution was only computed for the central 1⁰ × 1⁰ grid of the 4⁰ × 4⁰ gravity data grid in order to handle/reduce any edge effect. It is used as a terrain correction on the computed height anomaly. Furthermore, it is more significant in mountainous regions and relies heavily upon a detailed, accurate DEM (McCubbine et al., 2018). A computer program designed from *python* was used for this computation. Therefore substituting equations (5) and (6) into equation (4) to determine the gravity potential at a point P on the local vertical datum yields,

$$W_P = U_0 + \frac{\partial U_0}{\partial h}h_P + T_{GGM} + \frac{R}{4\pi}\iint_\sigma(\Delta g - \Delta g_{GGM} + G_1)\times S(\psi)\,d\sigma + T_{RTM} \tag{10}$$

Hence the gravity potential difference between global and local vertical reference (LLD) surfaces at a point P can be expressed as,

$$\delta W_P = W_0 - W_P = W_0 - [U_0 + \frac{\partial}{\partial}h_P + T_{GGM} + \frac{}{}\iint(\Delta g - \Delta g_{GGM} + G_1)\times S(\psi)\,d\sigma + T_{RTM}], \tag{11}$$

The h_P, in this case, refers to the ellipsoidal height at the tide gauge benchmark (TGBM). The height anomaly at the TGBM is estimated from Bruns's formula, and the gradient of the normal potential gives an approximation of the normal gravity value $\left(\frac{\partial U_0}{\partial h}\approx\ \gamma\right)$. Therefore, equation (11) can be expressed as:

$$\delta W_P = W_0 - W_P = (W_0 - U_0) + \ \gamma(h_P - H_P^{LLD} - \zeta_{GGM} - \zeta_{res} - \zeta_{RTM}), \tag{12}$$

where ζ_{GGM} gives the contribution of the GGM, expressed as,

$$\zeta_{GGM} = \zeta_0 + \frac{GM_g}{r\gamma}\sum_{n=2}^{n_{max}}\left(\frac{a_g}{r}\right)^n\sum_{m=0}^{n}\left(\Delta\bar{C}_{n,m}\cos m\lambda + \ \Delta\bar{S}_{n,m}\sin m\lambda\right)\times\bar{P}_{n,m}\left(\sin\bar{\varphi}\right) \tag{13}$$

where GM_g – is the gravity mass constant of the geopotential model in m^3s^{-2} defined from the geodetic model, γ – is normal gravity in ms^{-2}, r – is radial distance to the computational point in m, a_g – is the semi-major axis of the geopotential model, $\Delta\bar{C}_{n,m}$ – is the difference between the fully normalised harmonic coefficient $\bar{C}_{n,m}$ and harmonic coefficient generated by the normal gravity field $C^*_{n,m}$, $\Delta\bar{S}_{n,m}$ – is the difference between the fully normalised spherical harmonic coefficient $\bar{S}_{n,m}$ and the harmonic coefficient generated by the normal gravity field $S^*_{n,m}$, n and m are the degree and order for a geopotential model, $\bar{P}_{n,m}$ – is the fully normalised harmonics Legendre function, $\bar{\varphi}$ – is geocentric latitude of the computation point, λ – is the geodetic longitude of the computation point.

The ζ_0 represents a zero-degree harmonic term to the GGM geoid undulations with respect to a specific reference ellipsoid, $\zeta_0 = \frac{GM_g - GM_0}{R\gamma} - \frac{W_O - U_0}{\gamma}$ (Heiskanen & Moritz, 1967). The contribution of residual gravity anomalies (Δg) with the effect of the GGM and the terrain removed (ζ_{res}) is expressed as,

$$\zeta_{res} = \frac{R}{4\pi\gamma}\iint_\sigma(\Delta g - \Delta g_{GGM} + G_1)\times S(\psi)\,d\sigma \tag{14}$$

The contribution of the indirect effect on the height anomaly at the point $P(\zeta_{RTM})$ is given by Amos (2007) as,

$$\zeta_{RTM} = \frac{-\pi G\rho H_P^2}{\gamma} - \frac{G\rho R^2}{6\gamma}\iint_\sigma \frac{\left(H^3 - H_P^3\right)}{\ell^3} d\sigma \tag{15}$$

After estimation of the local gravity potential value W_P, using equation (10), the vertical datum offsets on the South African vertical datum in relation to the IHRS was computed as,

$$\delta\zeta_P = \frac{\delta W_P}{\gamma_P}. \tag{16}$$

This offset will provide an adjustment factor for the South African vertical datum to the IHRS. A unified vertical datum will provide a reference surface for engineering projects across countries, flooding control initiatives, plate tectonic movements determination and analysis, coastal hazard studies, unification of national gravity anomaly database, and improvement of the continental geoid, amongst other applications.

4 DATA AND METHODS

Several different data set were made available for the purpose of this study. The land and marine gravity data over South Africa were provided by the South African Council for Geoscience (SACGS) and Bureau Gravimetrique Internationale (BGI). However, the marine gravity data was coarse; it was supplemented with a global marine gravity model from CryoSat-2 and Jason-1 (Sandwell et al., 2014). Moreover, both the horizontal and vertical coordinates associated with the gravity data from SACGS and BGI are of low accuracy, as they have been interpolated from a 1:50000 map; this will introduce distortions on the resulting gravity anomalies.

The gravity data was screened for duplications using *Golden Surfer software*; the free-air gravity anomalies on the land gravity data range between −101.3 and 129.3 mGal with a mean and standard deviation of 16.3 and ±31.1 mGal, respectively. The free-air gravity anomalies from the land gravity data were compared to the set of free-air gravity anomalies generated using the GOCE-based GGM (TIM6) harmonic coefficients (up to degree and order 300). Thereafter, a mean difference of −2.1 mGal with a standard deviation of ±10.7 mGal was obtained.

The free-air gravity anomalies on the marine gravity data range between −97.5 and 115.7 mGal with a mean and standard deviation of 2.4 mGal and ±30.4 mGal, respectively. The free-air gravity anomalies from the marine gravity data were compared to the set of free-air gravity anomalies generated using the GOCE-based GGM (TIM6) harmonic coefficients (up to degree and order 300). Thereafter, a mean difference of 11.8 mGal with a standard deviation of ±17.5 mGal was obtained. The gravity data was limited to a 4º × 4º grid around each TGBM to reduce computation time, as depicted in Figure 2.

The first-order gravity data have a maximum uncertainty of ±1 mGal while the accuracy of the first-order levelling network in South Africa is estimated at $1.9\sqrt{L}mm$, L being the distance of a levelling line in km. The GPS measurements of the TGBMs were collected by the Nation Geo-Spatial Information (NGI), South African government agency. The heights were determined using differential carrier-phase GPS measurements linked to the national network of permanent GPS stations, TrigNet. The coordinates are in the ITRF2008(20016.2) reference frame and refer to the WGS84 ellipsoid. The internal accuracy of GPS coordinates is approximately ±1 and ±2 cm on the horizontal and vertical position, respectively (Odera, 2019). The differences between ellipsoidal and spheroidal heights are considered as height anomalies, as the South African LLD provides heights that are closer to the normal height system (Merry, 1985; Odera, 2019).

The SRTM data at 3 arc-second (90 m resolution) DEM was used for computation of the terrain effect (G_1 term). The DEM is uniform on the specified grid ($4^0 \times 4^0$) around each TGBM, as depicted in Figure 3 – Figure 6. The remove-restore method is used to compute the height anomalies of the TGBMs. The long-wavelength component of the disturbing potential was determined from the spherical harmonic coefficients of the latest GOCE-based GGM (TIM6 up to 300 degrees and order), and the medium wavelength component was determined from the gravity data residuals, using Stokes's integral as described in the previous subsection. The residual terrain model (RTM) was used to cater for the contribution of the short-wavelength component. A computer program designed from *python* was used for this computation. The four fundamental tide gauge benchmarks located in Cape Town (TGBM_CPT), Port Elizabeth (TGBM_PEL), East London (TGBM_ELN), and Durban (TGBM_DBN) over South Africa are shown in Figure 2.

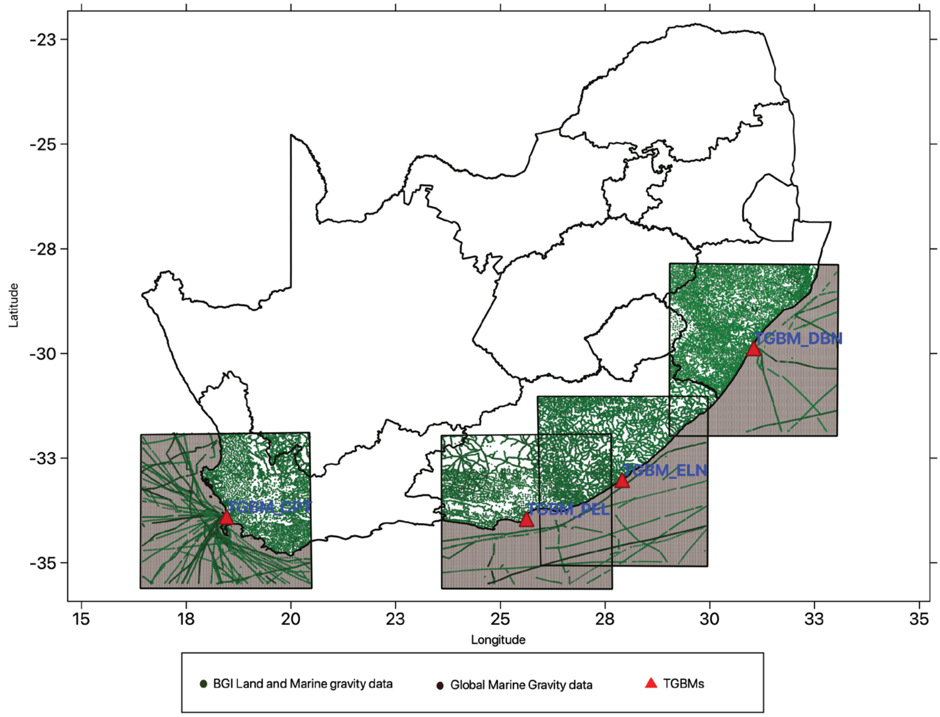

Figure 2: Distribution of the fundamental benchmarks over South Africa.

The elevation map round each TGBM was generated using DEM from SRTM90 to provide a terrain visualisation, as depicted in Figure 3 - Figure 6. A kriging interpolation method was used to generate contour maps because it is statistically more sophisticated and it allows identifying distortions in the data. Moreover, it was used to evaluate the contribution of the indirect effect on the height anomaly.

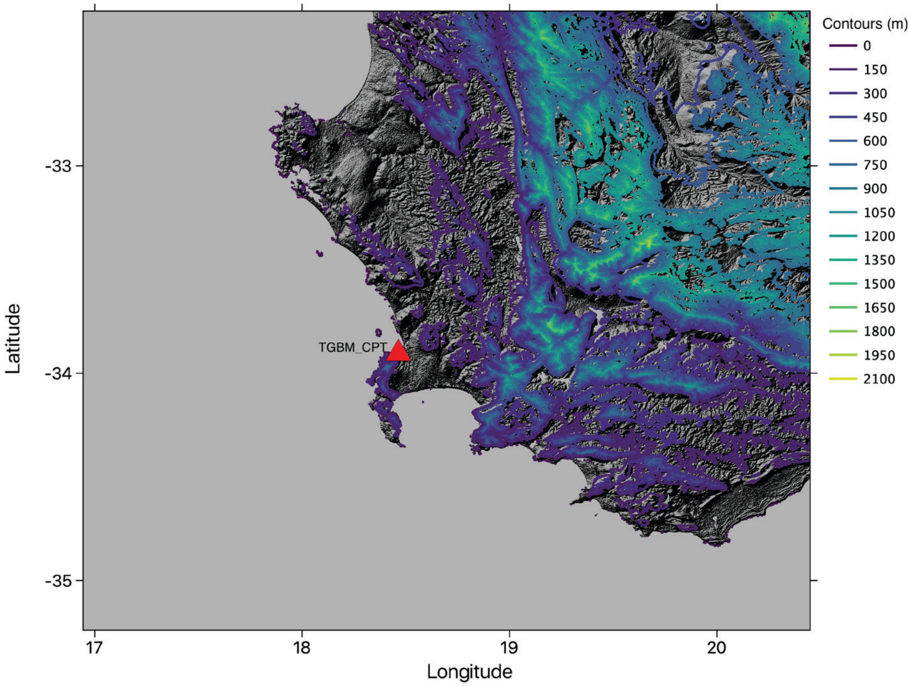

Figure 3: Elevation around Cape Town TGBM.

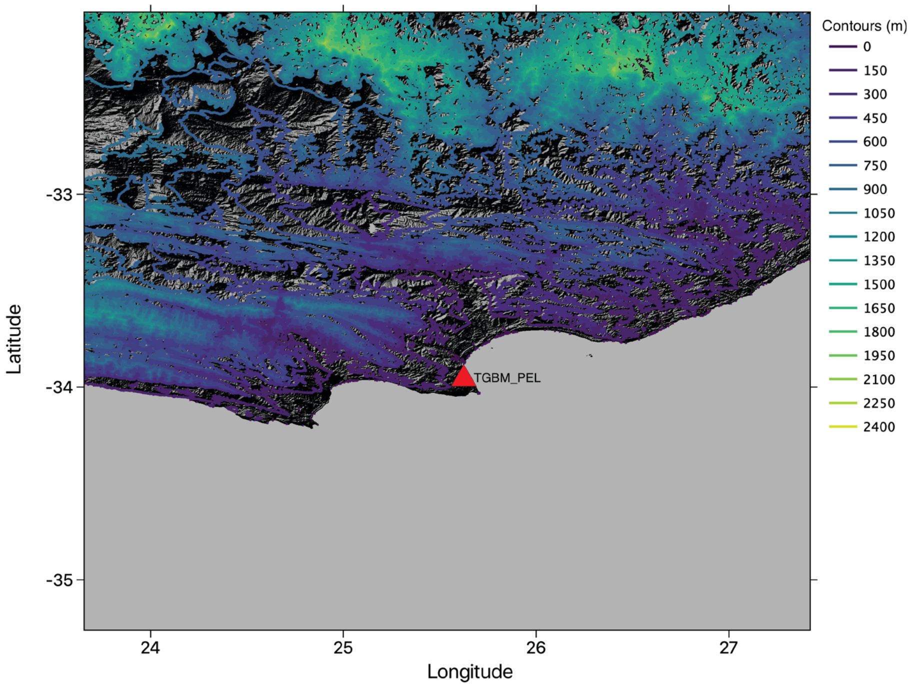

Figure 4: Elevation around Port Elizabeth TGBM.

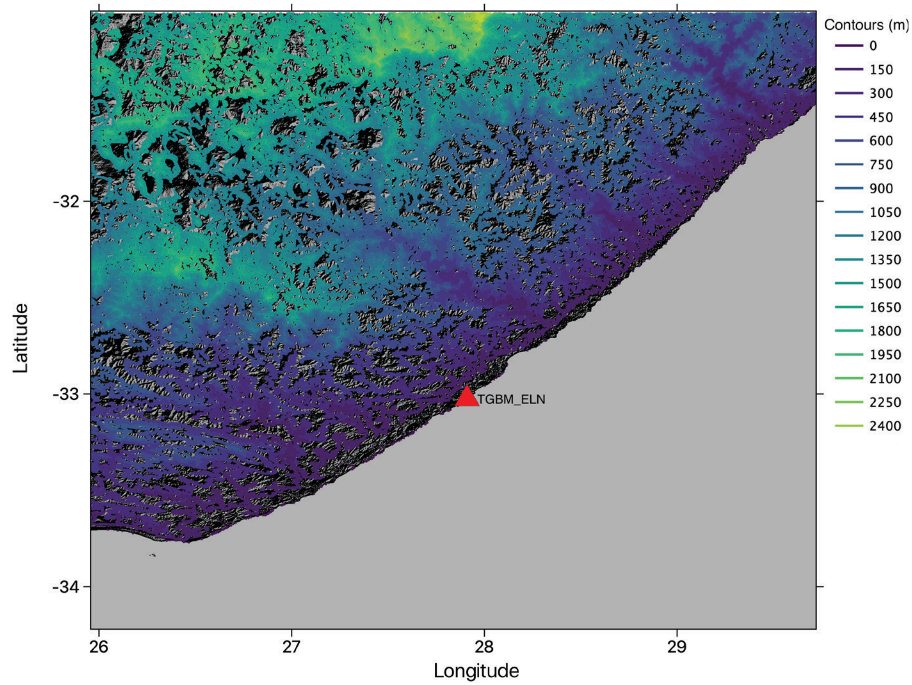

Figure 5: Elevation around East London TGBM.

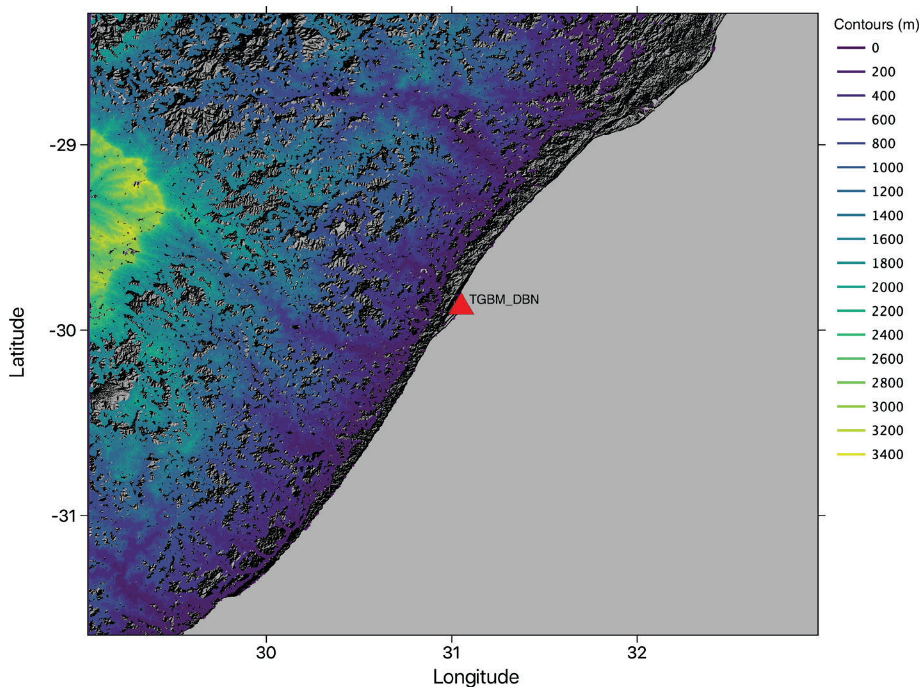

Figure 6: Elevation around Durban TGBM.

5 RESULTS AND DISCUSSION

The derived physical constant of the normal gravity potential for the WGS84 reference ellipsoid is U_0 = 62636851.7146 m^2s^{-2} as given by the International Earth Rotation and Reference Systems Service (IERS). The gravity residuals used on Stokes integral, as expressed in equation (6), was determined from the gravity anomalies computed from the observed gravity data (Δg), the gravity anomalies generated by the coefficients of the spherical harmonics (Δg_{GGM}), from the GOCE based GGM and the Molodensky G_1 term determined from a convolution of heights with gravity anomalies. The residual gravity anomalies around each TGBM are as depicted in Figure 7 – Figure 11. A kriging gridding method on the *Golden Sufer software* was used to generate the contour maps, it produces a more accurate grid file, and it is a very flexible gridding method.

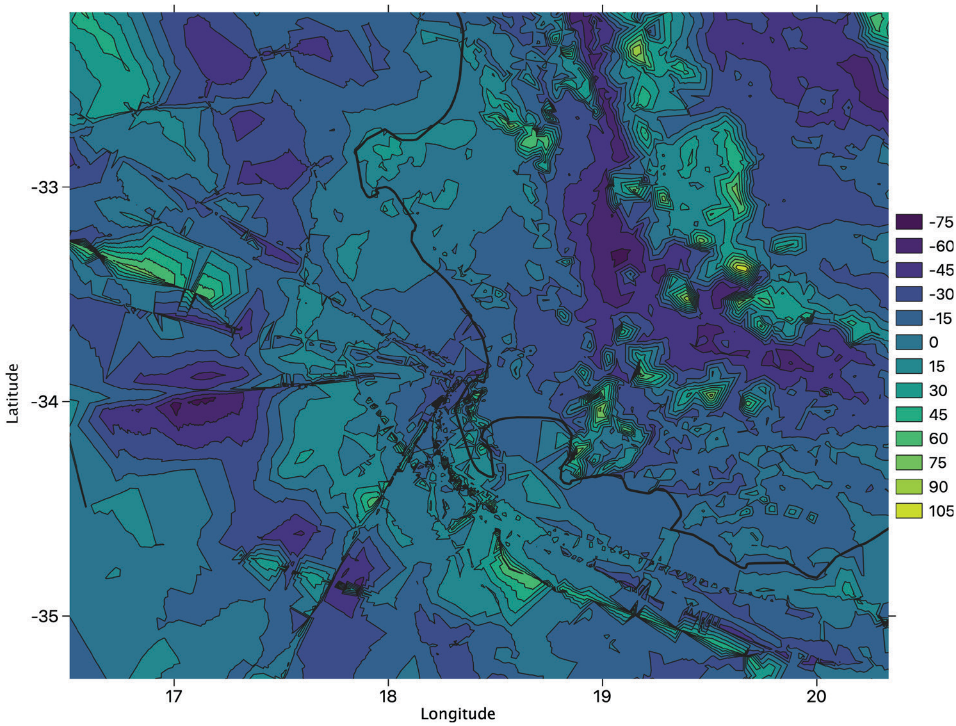

Figure 7: Residual gravity anomalies around Cape Town TGBM (units are in mGal).

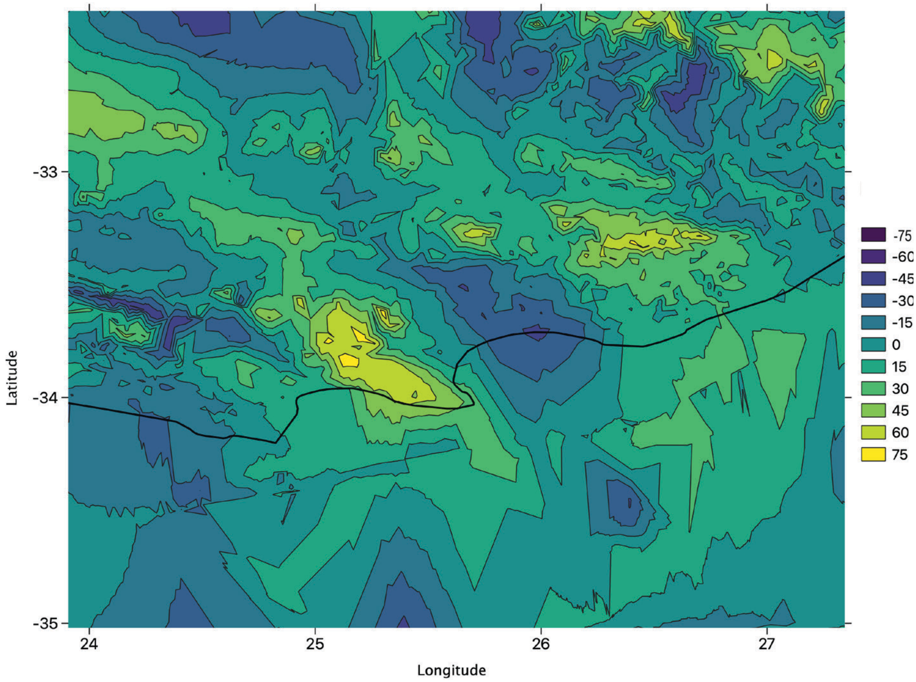

Figure 8: Residual gravity anomalies around Port Elizabeth TGBM (units are in mGal).

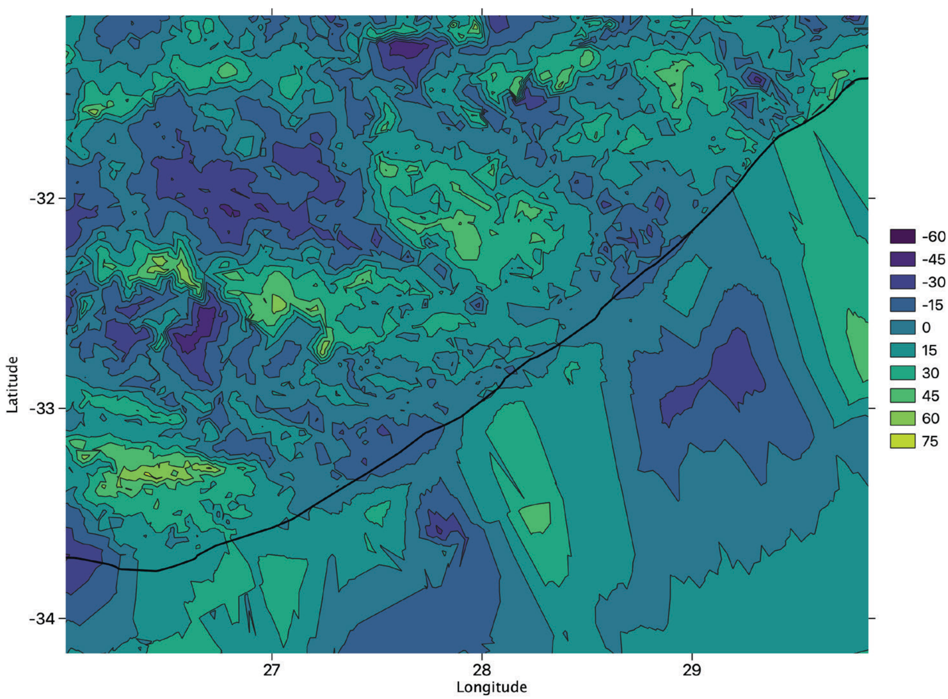

Figure 9: Residual gravity anomalies around East London TGBM (units are in mGal).

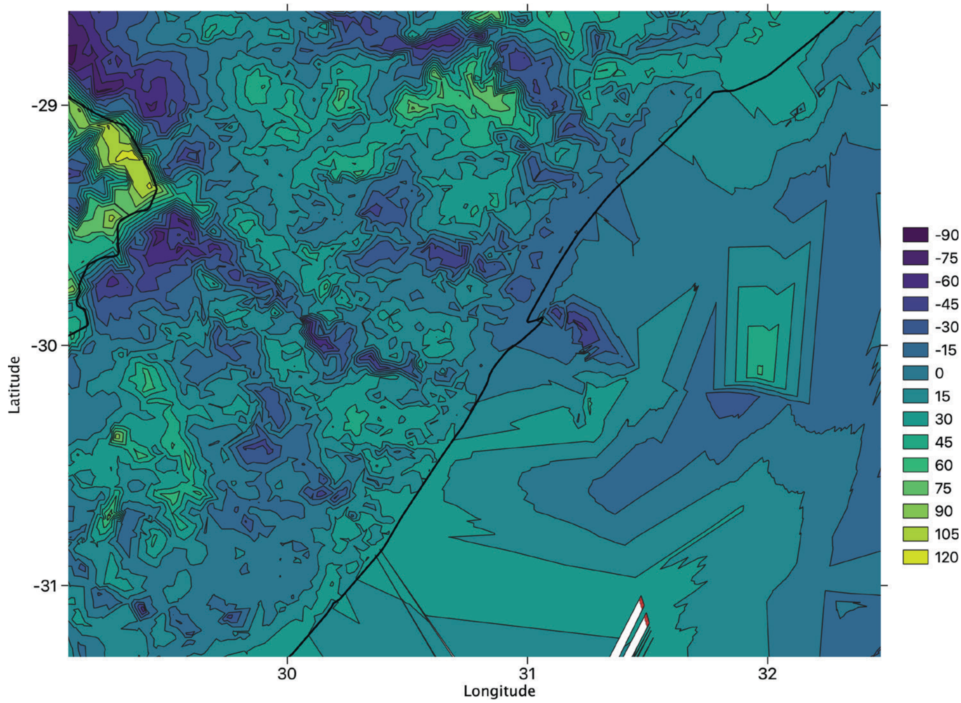

Figure 10: Residual gravity anomalies around Durban TGBM (units are in mGal).

As discussed in the previous section that the South African vertical datum is constrained to four TGBMs, the vertical datum offset is determined on the four fundamental benchmarks in relation to the IHRS, the estimated potential discrepancies are as depicted in Figure 11. The vertical datum offset at each TGBM was evaluated using equation (16), and the potential difference between the local and the global reference surface was evaluated using equation (12). The components involved in the computation of the vertical datum offset at each TGBM, are as illustrated in Table 1. Results of estimated offsets are also included in Table 1.

Table 1: Vertical datum offset parameters and estimated offset at each TGBM.

TGBM	h_P(m)	H_P^{LLD}(m)	ζ_{GGM}(m)	ζ_{res}(m)	ζ_{RTM}(m)	W_P(m²s⁻²)	δW_P(m²s⁻²)
CPT	34.423	3.6281	31.996	0.085	-1.519	62636852.811	0.589
PEL	31.487	4.2233	29.276	0.016	-1.997	62636855.393	-1.993
ELN	33.823	4.4153	30.642	0.018	-1.160	62636855.993	-2.593
DBN	32.678	4.3076	28.465	-0.010	-0.477	62636851.246	2.154

The gravity potential at each TGBM in South Africa deviates from the potential of the global reference surface by 0.589, −1.993, −2.593 and 2.154 m^2s^{-2} for Cape Town, Port Elizabeth, East London and Durban, respectively. These deviations are as depicted in Figure 11.

The corresponding vertical datum offset between the international height reference system and the four fundamental benchmarks over South Africa are 6.013, −20.347, −26.478, and 21.996 cm for Cape Town, Port Elizabeth, East London and Durban, respectively. These offsets can be used for the unification of the South African datum at the four TGBMs in a manner that is consistent with the international height reference system. The estimated gravity potential on the four fundamental benchmarks are as illustrated in Table 1.

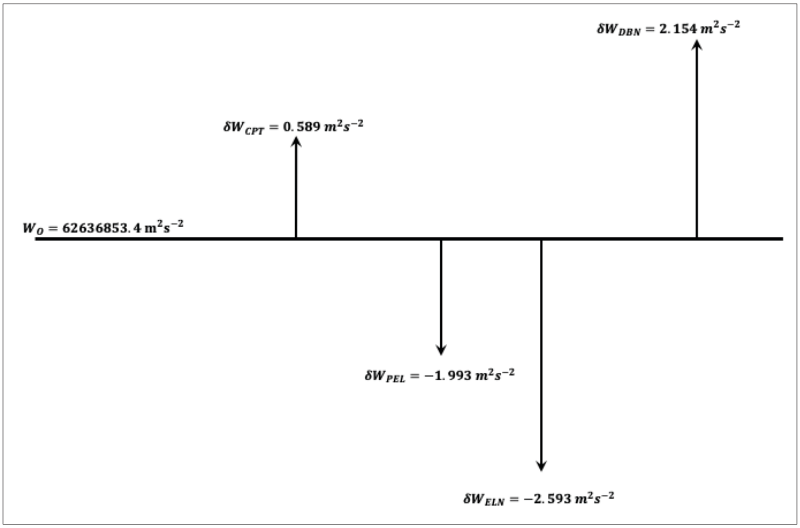

Figure 11: Vertical datum offset on the four TGBM in relation to the global vertical datum.

This forms part of the datum parameters, and it should be as reliable as possible. The quality of the fundamental benchmarks can be improved by being connected to the gravity data networks. The desired physical heights system can be deduced from geopotential values using equation (3). The advantages of using geopotential value for height determination is that there is no need to compute orthometric or normal corrections to the measured height differences, thus avoiding any approximations in the corrections and it is very easy to convert between height systems, as one does not have to compute a new set of corrections.

6 CONCLUSION

The vertical datum offset on the South African vertical datum in relation to the IHRS, has been estimated using the single-point-based GBVP solution at four TGBMs. The gravity data on a 4º × 4º grid around each fundamental benchmark was selected for the purpose of estimating their disturbing potential; this was performed in combination with the spherical harmonics coefficients from the GOCE based GGM, TIM-R6 (complete to 300 degrees and order).

The gravity potential at each TGBM in South Africa deviates from the potential of the global reference surface by 0.589, −1.993, −2.593 and 2.154 m^2s^{-2} for Cape Town, Port Elizabeth, East London and Durban, respectively. The corresponding vertical datum offset between the international height reference system and the four fundamental benchmarks over South Africa are 6.013, −20.347, −26.478, and 21.996 cm for Cape Town, Port Elizabeth, East London and Durban, respectively. This evaluation provides South Africa with a direct link to the IHRS and a positive step towards the South African vertical datum realisation and unification.

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