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DEFORMACIJSKA ANALIZA PO POSTOPKU CASPARY

DEFORMATION ANALYSIS: THE CASPARY APPROACH

Veton Hamza, Bojan Stopar, Tomaž Ambrožič

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IZVLEČEK

V članku je opisan postopek Caspary, ki je eden izmed postopkov deformacijske analize. Značilnosti tega postopka so testiranje skladnosti geodetske mreže, določitev stabilnosti točk med dvema terminskima izmerama, transformacija geodetske mreže s transformacijo S, izračun premikov in grafična predstavitev vektorjev premikov. V članku je najprej podano teoretično ozadje postopka, nato je postopek uporabljen na primeru simuliranih meritev dveh terminskih izmer. Rezultati postopka Caspary na obravnavanem primeru ne odstopajo bistveno od rezultatov, dobljenih s postopki Hannover, Karlsruhe, Delft, Fredericton, München in z robustnimi metodami.

ABSTRACT

In this paper, the theoretical background of the Caspary method of geodetic deformation analysis is described and implemented in a simulated geodetic network in which two epochs of measurements are used. The Caspary approach foresees congruence testing of the geodetic network, the determination of the stable points between two analysed epochs, the transformation of the geodetic network using S-transformation, the calculation of displacements, and, in the last step, the graphical presentation of displacement vectors. Results obtained from the presented example are similar to those presented in the Hannover, Karlsruhe, Delft, Fredericton, München, and robust methods.

KLJUČNE BESEDE

postopek Caspary, geodetska mreža, deformacijska analiza, meritve, računski primer

KEY WORDS

Caspary approach, geodetic network, deformation analysis, observations, numerical example

1 INTRODUCTION

The Caspary approach was developed at the University of New South Wales in Australia by Caspary (1988). This methodology foresees to use geodetic observations that are carried out in two epochs; it also includes geological and geophysical analyses in order to have more information about the stability of the reference points (Mihailović and Aleksić, 1994). These analyses can obtain information's for the point's stability, but statistical testing does the verification of point's stability between two epochs of observations that are analysed. Caspary approach has similarities with Hannover method in defining stable points while it foresees to transform the geodetic datum by S-transformation and use as datum points only those points that have remained stable and are confirmed by statistical tests (Mihailović and Aleksić, 1994).

In general, methods that are using geodetic observations for estimating displacements contain four steps that are also implemented in the Caspary approach (Caspary, 1988; Mihailović and Aleksić, 1994):

1. Firstly, the geodetic network needs to be established, after the accuracy that needs to be achieved, and the observation plan has to be defined.
2. In the second step, the network adjustment is carried out for each epoch separately, outliers are removed, and point coordinates are calculated.
3. Points from the reference block that have been shifted between two epochs are confirmed in the third step; these points are not used as datum points in the upcoming computations.
4. In the last step, after S-transformation displacements are estimated for both object and reference points, error ellipses and displacement vectors are shown graphically.

2 THEORETICAL BACKGROUNDS

2.1 Establishment of the geodetic network and definition of the observation plan

The geometry of the geodetic network depends on the terrain configuration, type and size of the object that will be monitored and the ability of the surveyor to establish such geodetic network in certain terrain conditions that will be used after to monitor the object stability regarding the projected accuracy (Mihailović and Aleksić, 2008). Observation plan needs to be determined in such a way that it can be realized taking into account the terrain configuration. Aiming to define the geometry of the geodetic network and the observation plan, the optimisation of the first order is used while the weights of planned measurements are defined in the optimisation of the second order.

2.2 Geodetic network adjustment and outlier detection

Caspary approach is classified in the group of methods that analyse two epochs of observations, and the coordinate differences between two epochs are presented as displacements. Firstly, the accuracy of measurements should be tested; homogenous accuracy is achieved in case that the a posteriori variance is statistically equal in both of epochs, which is tested with the following hypothesis (Ambrožič, 2001; Caspary, 1988; Mihailović and Aleksić, 1994):

$$H_0 : E(\hat{\sigma}_{01}^2) = (\hat{\sigma}_{02}^2) = \sigma_0^2 , \tag{1}$$

$$H_a : E(\hat{\sigma}_{01}^2) \neq (\hat{\sigma}_{02}^2) \neq \sigma_0^2 . \tag{2}$$

Verification of null hypothesis is done by the following *T*-test that belongs to the *F*-distribution:

$$T = \frac{\hat{\sigma}_{01}^2}{\hat{\sigma}_{02}^2} \leq F_{1-\alpha, f_1, f_2}, \tag{3}$$

$$f_1 = n_1 - u_1 + d_1, \tag{4}$$

$$f_2 = n_2 - u_2 + d_2, \tag{5}$$

α – risk level,

f_i – redundancy number (degrees of freedom) for certain epoch,

n_i – number of observations for certain epoch,

u_i – number of unknowns for certain epoch,

d_i – datum defect for certain epoch.

In case that the null hypothesis isn't rejected, then the common variance need to be computed (Caspary, 1988; Mihailović and Aleksić, 1994):

$$s^2 = \frac{f_1\hat{\sigma}_{01}^2 + f_2\hat{\sigma}_{02}^2}{f_1 + f_2} = \frac{q}{f}. \tag{6}$$

The Caspary approach foresees to use inner constrained datum definition, during adjustment computation of the geodetic network; the following conditions need to be fulfilled (Kuang, 1996; Caspary, 1988):

$$\mathbf{v}_i^{\mathrm{T}}\mathbf{P}_i\mathbf{v}_i = min., \tag{7}$$

$$\mathbf{x}_i^{\mathrm{T}}\mathbf{x}_i = min. \text{ and } \hat{\mathbf{x}}_i = \mathbf{x}_{0i} + \mathbf{x}_i, \tag{8}$$

$\mathbf{v}_i$ – vector of residuals for certain epoch,

$\mathbf{P}_i$ – weight matrix for certain epoch,

$\mathbf{x}_{0i}$ – vector of approximate coordinates for certain epoch,

$\mathbf{x}_i$ – vector of parameter corrections for certain epoch,

$\hat{\mathbf{x}}_i$ – vector of adjusted coordinates for certain epoch.

The geodetic network is adjusted as free network in both epochs, the orientation unknowns (and the potential unknown of the scale factor) need to be eliminated by using the Gauss or other methods (Mihailović and Aleksić, 1994; Mihailović, 1981). Baarda's Data Snooping, Pope's Tau Method or Danish Method can be implemented to detect outliers (Caspary, 1988; Grigillo and Stopar, 2003) and the accuracy of measured angles and distances need to be harmonized before the adjustment of the geodetic network (Ambrožič, 2004).

2.3 Detection of unstable point by Caspary approach

Caspary approach has similarity with some of the other methods used in deformation analyses in the process of determining stable points from the reference block, especially with the method of Hannover (Mihailović and Aleksić, 1994). Firstly, after adjusting the geodetic network as free network the vector Δ of adjusted coordinate differences and the pseudo inverse matrix of normal equations $\mathbf{Q}_{\Delta\Delta}^{-}$ are defined as follows (Caspary, 1988):

$$\Delta = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1, \tag{9}$$

$$\mathbf{Q}_{\Delta\Delta}^{-} = \mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}1}^{+} + \mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}2}^{+} = \mathbf{B}_1^{\mathrm{T}}\mathbf{P}_1\mathbf{B}_1 + \mathbf{B}_2^{\mathrm{T}}\mathbf{P}_2\mathbf{B}_2, \tag{10}$$

$\mathbf{B}_i$ – coefficient matrix of observations for certain epoch.

The vector Δ (9) need to be decomposed in two sub vectors $\Delta^{\mathrm{T}} = (\Delta_n^{\mathrm{T}}\ \Delta_p^{\mathrm{T}})$ where Δ_n^{T} presents points from the reference block that will be tested for their stability and Δ_p^{T} presents object points that are treated as unstable points in all steps of the upcoming computations:

$$\Delta = \begin{bmatrix} \Delta_n \\ \Delta_p \end{bmatrix}. \tag{11}$$

The same logic is followed for decomposing matrix (Caspary, 1988):

$$\mathbf{Q}_{\Delta\Delta}^{-} = \mathbf{P}\Delta\Delta = \begin{bmatrix} \mathbf{P}_{nn} & \mathbf{P}_{np} \\ \mathbf{P}_{pn} & \mathbf{P}_{pp} \end{bmatrix}. \tag{12}$$

In order to define if points have been shifted between two epochs, the following hypothesis are set (Caspary, 1988):

$$H_0 : E\left(\hat{\mathbf{x}}_{n1}\right) = \left(\hat{\mathbf{x}}_{n2}\right) = \hat{\mathbf{x}}_n, \tag{13}$$

$$H_a : E\left(\hat{\mathbf{x}}_{n1}\right) \neq \left(\hat{\mathbf{x}}_{n2}\right) \neq \hat{\mathbf{x}}_n. \tag{14}$$

Not rejection of null hypothesis means that all tested points have remained stable while in the opposite case, there are some points that have changed their position between two epochs. Hypothesis defined in (13) and (14) belongs to the *F*-distribution, and they are verified by the following congruency test (Caspary, 1988):

$$T = \frac{\left(q_\Delta\right)_n / f_\Delta}{q / f} = \frac{\left(q_\Delta\right)_n / f_\Delta}{S^2} \sim F_{(1-a, f_\Delta, f)}, \tag{15}$$

whereas:

$$\left(q_\Delta\right)_n = \Delta_n^{\mathrm{T}}\mathbf{Q}_{nn}^{-}\Delta_n, \tag{16}$$

$$f_\Delta = 2m - d, \tag{17}$$

m – number of points that are tested for their stability.

The matrix $\mathbf{Q}_{nn}^{-}$ is estimated as follows (Caspary, 1988):

$$\mathbf{Q}_{nn}^{-} = \mathbf{P}_{nn} - \mathbf{P}_{np}\mathbf{P}_{pp}^{-1}\mathbf{P}_{pn}. \tag{18}$$

In case that of all points of the reference network are tested, and the network is not divided into reference block and object block then the matrix $\mathbf{Q}_{nn}^{-}$ (18) is equal to the matrix $\mathbf{Q}_{\Delta\Delta}^{-}$ (12). The value of the T-test defined by the equation (15) need to be compared with the corresponding value F of the F-distribution $F_{(1-a,\, f_\Delta,\, f)}$, if $T \le F_{(1-a,\, f_\Delta,\, f)}$ then the null hypothesis is not rejected, in case that $T > F_{(1-a,\, f_\Delta,\, f)}$ the null hypothesis is rejected. Rejection of the null hypothesis means that there are reference points than have been shifted, and further analyses need to take place in order to detect unstable points.

The process of detecting the unstable points is executed iteratively. Another decomposition of the vector Δ and matrix $\mathbf{Q}_{\Delta\Delta}^{-}$ is performed following the same logic as in the equations (11) and (12). The sub-vector Δ_n represents all points from the reference block that are tested for stability, for which the assumption that they are stable is considered in the process of defining stable points while sub-vector Δ_p represents only one point from the reference block in each of the iterations (Caspary, 1988). The same rule is followed for the decomposition of the matrix $\mathbf{Q}_{\Delta\Delta}^{-}$, the $\mathbf{P}_{nn}$ block belongs to all points from the reference block while the $\mathbf{P}_{pp}$ block belongs to one point. The quadratic form $(q_\Delta)_n$ (16) need to be decomposed in two subforms after transformation of the vector Δ that is carried out as follows (Caspary, 1988):

$$\begin{bmatrix} \overline{\Delta}_n \\ \overline{\Delta}_p \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{P}_{pp}^{-1}\mathbf{P}_{pn} & \mathbf{I} \end{bmatrix} \Delta. \tag{19}$$

The sub-vector $\overline{\Delta}_p$ is referring to tested point and sub-vector $\overline{\Delta}_n$ is referring to all remain points from reference block, in order to define the value of $(q_\Delta)_p$ for each point separately in all iterations, this decomposition is needed. As an unstable point will be declared the point that will have a greater value of $(q_\Delta)_p$ in each of iterations, the value of $(q_\Delta)_p$ is estimated by the following expression (Caspary, 1988):

$$\left(q_\Delta\right)_p = \overline{\Delta}_p^{\mathbf{T}} \mathbf{P}_{pp} \overline{\Delta}_p . \tag{20}$$

In the next iteration, the already defined unstable point will be in the same group with the object points, the decomposition of the vector Δ (11) and the matrix $\mathbf{Q}_{\Delta\Delta}^{-}$ (12) will continue iteratively until all the shifted points are detected. Congruency test defined in (15) is repeated until the null hypothesis (13) is not rejected, which means that remained points are stable.

2.4 Estimation of displacements

Displacements are estimated after all shifted points are detected, the vector Δ and the matrix $\mathbf{Q}_{\Delta\Delta}^{-}$ are partitioned, as is shown (Caspary, 1988):

$$\Delta = \begin{bmatrix} \Delta_r \\ \Delta_o \end{bmatrix}, \tag{21}$$

$$\mathbf{Q}_{\Delta\Delta}^{-} = \mathbf{P}_{\Delta\Delta} = \begin{bmatrix} \mathbf{P}_{rr} & \mathbf{P}_{ro} \\ \mathbf{P}_{or} & \mathbf{P}_{oo} \end{bmatrix}. \tag{22}$$

The sub-vector Δ_r represents stable points while the sub-vector Δ_o represents all shifted points; for the decomposition of $\mathbf{Q}_{\Delta\Delta}^{-}$ matrix the same logic is followed, coefficients from $\mathbf{P}_{rr}$ block belongs to stable points while $\mathbf{P}_{oo}$ block belongs to object points and unstable reference points. In the last transformation, points that have remained stable will define the geodetic datum. S-transformation is implemented to transform the geodetic datum in stable points for each epoch separately in order to not carried out another adjustment. Vector Δ and matrix $\mathbf{Q}_{\Delta\Delta}$ need to be transformed into the new geodetic datum defined by stable points with the following equations (Stopar and Marjetič, 2007):

$$\Delta_{stab} = \mathbf{S}_{stab}\Delta, \quad (23)$$

$$\mathbf{Q}_{\Delta\Delta stab} = \mathbf{S}_{stab}\mathbf{Q}_{\Delta\Delta}\mathbf{S}^{\mathrm{T}}_{stab}, \quad (24)$$

$$\mathbf{S}_{stab} = \mathbf{I} - \mathbf{H}(\mathbf{H}\mathbf{E}_{stab}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{E}_{stab}. \quad (25)$$

In the matrix $\mathbf{E}_{stab}$ all elements are equal to null except diagonal elements of the stable points that are equal to one. **I** matrix is the identity matrix, and the **H** matrix is defined according to the datum defect of the geodetic network (Krüger, 1980). Displacements of the object points (shifted points) are estimated by the following equation (Caspary, 1988):

$$\overline{\Delta}_o = \mathbf{P}_{oo}^{-1}\mathbf{P}_{or}\Delta_r + \Delta_o. \quad (26)$$

The Caspary approach foresees to show displacements also graphically, vectors of displacements and error ellipses (confidence ellipses/deformation ellipses) with a significance level of 95% need to be estimated (Caspary, 1988). Coordinates of the first (null) epoch are defining the origin of error ellipses while coordinate differences between first and second epoch are presenting the length of displacements vectors. In case that the vector will be out of the error ellipse that point will be reconfirmed as unstable, in stable point the vector of displacement need to be inside the error ellipse. Equations that are used to estimate the error ellipses from the matrix $\mathbf{Q}_{\Delta\Delta stab} = \mathbf{Q}_{\Delta\Delta stab1} + \mathbf{Q}_{\Delta\Delta stab2}$ are define as follows (Caspary, 1988):

$$a = s\sqrt{\lambda_1 2F_{1-\alpha,2,f}}, \quad (27)$$

$$b = s\sqrt{\lambda_2 2F_{1-\alpha,2,f}}, \quad (28)$$

$$\tan 2\Theta = \frac{2q_{\hat{x}\hat{y}}}{q_{\hat{x}\hat{x}} - q_{\hat{y}\hat{y}}}, \quad (29)$$

$$\lambda_1 = \frac{1}{2}\left(q_{\hat{x}\hat{x}} + q_{\hat{y}\hat{y}} + z\right), \quad (30)$$

$$\lambda_2 = \frac{1}{2}\left(q_{\hat{x}\hat{x}} + q_{\hat{y}\hat{y}} - z\right), \quad (31)$$

$$z^2 = \left(q_{\hat{x}\hat{x}} - q_{\hat{y}\hat{y}}\right)^2 + 4q_{\hat{x}\hat{y}}q_{\hat{x}\hat{y}}. \quad (32)$$

Graphic interpretation of the obtained results is the last step of the Caspary approach that is carried out after estimation of the error ellipses defined by the presented equations (27–32).

3 PRACTICAL EXAMPLE

The Caspary method was tested in simulated geodetic network consisted of 7 points, 24 horizontal directions and 24 distances. The a priori variance for the directions is σ_{si} = 1" and the a priori variance for the distances is equal to σ_{di} = 5 mm. The plan of observations is the same in both epochs while previously the geodetic network was tested by different researchers with other methods used in deformation analyses such are:

- Hannover (Ambrožič, 2001),
- Karlsruhe (Ambrožič, 2004),
- Delft (Marjetič, Zemljak and Ambrožič, 2013),
- Fredericton (Vrečko and Ambrožič, 2013),
- München (Soldo and Ambrožič, 2018),
- Robust methods (Ambrožič et al., 2019).

Caspary approach foresees to fulfil the conditions defined in equation (7) and (8) during the adjustment of the network, regarding this the cofactor matrix of unknowns is calculated by the following expression (Krüger, 1980):

$$\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = (\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{B} + \mathbf{H}\mathbf{H}^{\mathrm{T}})^{-1} - \mathbf{H}\mathbf{H}^{\mathrm{T}}. \tag{33}$$

The datum matrix $\mathbf{H}^{\mathrm{T}}$ matrix depends on the observations that are carried out, and it is defined according to the defect of datum equal to three in both epochs. More information for datum definition can be founded in (Krüger, 1980). Adjustment computation of the geodetic network was carried out for each epoch separately, and results are presented in Table 1.

Table 1: Results from the adjustment computation.

	First epoch	Second epoch
	$i = 1$	$i = 2$
$\hat{\sigma}_{0i}^2$	0.941	1.337
n_i	48	48
u_i	14 + 7	14 + 7
d_i	3	3
f_i	30	30
s^2; equation (6)	1.337	
f; equation (6)	60	

From the results presented in Table 1, it can be concluded that homogenous accuracy has been reached in both of epochs, the value of common variance s^2 was used in further calculations. The vector Δ and the matrix $\mathbf{Q}_{\Delta\Delta}^{-}$ were estimated regarding equations (9) and (10), respectively. Taking into account that the geodetic network is not divided into reference block and object block the matrix $\mathbf{Q}_{nn}^{-}$ is equal to the matrix $\mathbf{P}_{nn}$. The next step was to test points stability between two epochs regarding hypothesis defined in (13) and (14) while to check hypothesis the congruency test presented in equation (15) was used. The statistical test has shown that the null hypothesis (13) is rejected and results from this calculation are presented in Table 2.

Table 2: Results from the congruency test.

Parameter	m	f_Δ	$(q_\Delta)_n$	T	$F_{(0.975,11,60)}$
Values	7	11	1772.21	141.48	1.95

The congruency test has confirmed that some points from the geodetic network have been shifted between two epochs. Results from the computations regarding the explanations from section 2.3 are summarized in Table 3 while results from the statistical congruency test (15) in each of the iterations are shown in Table 4.

Table 3: Localization of unstable points.

Point	1st iteration	2nd iteration	3rd iteration	4th iteration
	$(q_\Delta)_p$	$(q_\Delta)_p$	$(q_\Delta)_p$	$(q_\Delta)_p$
1	**755.51**			
2	562.59	321.26	**505.70**	
3	415.29	347.99	394.94	**146.15**
4	94.53	99.03	52.00	75.72
5	67.62	75.56	17.13	3.87
6	8.99	95.81	52.00	0.51
7	664.76	**363.61**		

Table 4: Results from the statistical test (15) and critical values.

Parameter	1st iteration	2nd iteration	3rd iteration	4th iteration	5th iteration
m	7	6	5	4	3
f_Δ	11	9	7	5	3
T	141.48	99.20	81.93	25.89	0.36
F	1.95	2.04	2.17	2.37	2.76

From the presented results, it was concluded that point 1, 2, 3 and 7 had been shifted while point 4, 5 and 6 have remained stable. In the fifth iteration, the congruency test has confirmed that the null hypothesis (13) isn't rejected and in the following step the defined stable points have been used as datum points to carried out the S- transformations presented with equations (23), (24) and (25). Displacements are calculated after the transformation of the geodetic datum, error ellipses and displacement values through Y and X-axis are presented in Table 5 while error ellipses and displacements vectors are graphically shown in Figure 1.

Table 5: Displacements and error ellipses.

Point	Displacements		Error ellipses		
	d_y [mm]	d_x [mm]	a [mm]	b [mm]	Θ [°]
1	– 19.2	– 37.9	11.1	9.2	104
2	– 38.4	49.4	13.2	10.4	62
3	20.8	– 43.9	11.9	9.1	2
4	–	–	6.4	3.7	74
5	–	–	6.2	5.4	159
6	–	–	6.5	2.9	55
7	23.9	43.1	7.2	6.7	151

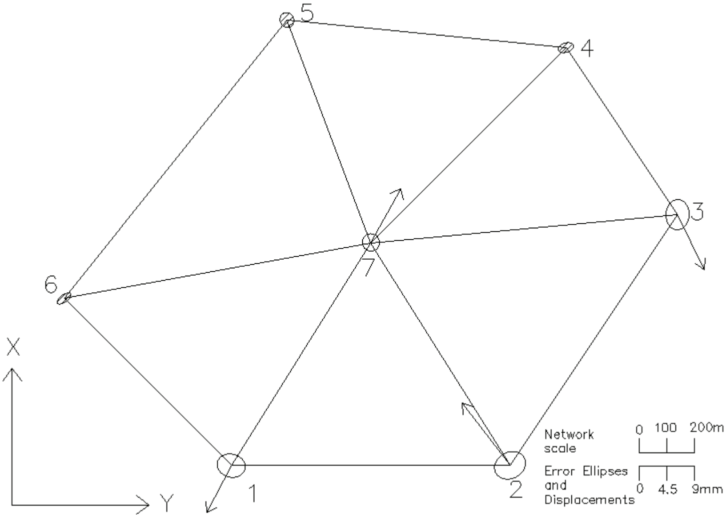

Figure 1: Error ellipses and displacements.

4 COMPARISON OF RESULTS FROM CASPARY METHOD WITH OTHER METHODS

Results from the Caspary method are compared with the results from the method of Hannover, Karlsruhe, Delft, Fredericton and München approach and presented in Table 6. Point 4, 5 and 6 are defined as stable points while point 1, 2, 3 and 7 are confirmed as unstable points in all of the methods. Slight difference in results between methods was noticed, but the estimated values of displacements are very similar to simulated displacements.

Table 6: Simulated displacements and results from deformation analyses by Hannover, Karlsruhe, Delft, Fredericton, München and Caspary approach.

Point		**1**	**2**	**3**	**4**	**5**	**6**	**7**
Simulated	d_y [mm]	– 20.0	– 30.0	25.0	0.0	0.0	0.0	25.0
	d_x [mm]	– 34.6	52.0	– 43.3	0.0	0.0	0.0	43.3
	d [mm]	40.0	60.0	50.0	0.0	0.0	0.0	50.0
	υ [°]	210	330	150	–	–	–	30
Hannover	d_y [mm]	– 19.6	– 38.7	20.6	– 4.0	– 6.4	3.3	23.6
	d_x [mm]	– 38.0	49.0	– 44.3	5.1	– 7.1	– 10.6	42.9
	d [mm]	42.8	62.4	48.9	6.5	10.0	11.1	49.0
	υ [°]	207	322	155	322	222	163	29
	Movement	yes	yes	yes	no	no	no	yes
Karlsruhe	d_y [mm]	– 19.7	– 38.8	20.6	–	–	–	23.6
	d_x [mm]	– 38.0	49.0	– 44.4	–	–	–	42.9
	d [mm]	42.8	62.5	48,9	–	–	–	49,0
	υ [°]	207	322	155	–	–	–	29
	Movement	yes	yes	yes	no	no	no	yes

Point		1	2	3	4	5	6	7
Delft	d_y [mm]	– 19.4	– 38.1	21.4	0.7	– 0.8	0.0	24.0
	d_x [mm]	– 37.5	49.5	– 43.5	1.0	– 2.3	1.3	42.9
	d [mm]	42.2	62.5	48.5	1.2	2.4	1.3	49.2
	υ [°]	207	322	154	35	199	0	29
	Movement	yes	yes	yes	no	no	no	yes
Fredericton	d_y [mm]	– 19.6	– 38.7	20.6	–	–	–	23.6
	d_x [mm]	– 38.0	49.0	– 44.3	–	–	–	42.9
	d [mm]	42.8	62.5	48.9	–	–	–	48.9
	υ [°]	207	322	155	–	–	–	29
	Movement	yes	yes	yes	no	no	no	yes
München	d_y [mm]	– 19.5	– 38.2	21.4	0.7	– 0.8	0.0	24.0
	d_x [mm]	– 37.6	49.5	– 43.6	1.0	– 2.2	1.4	42.9
	d [mm]	42.4	62.5	48.6	1.2	2.3	1.4	49.2
	υ [°]	207	322	154	35	200	0	29
	Movement	yes	yes	yes	no	no	no	yes
Caspary	d_y [mm]	– 19.2	– 38.4	20.8	–	–	–	23.9
	d_x [mm]	– 37.9	49.4	– 43.9	–	–	–	43.1
	d [mm]	42.5	62.5	48.6	–	–	–	49.3
	υ [°]	207	322	154	–	–	–	29
	Movement	yes	yes	yes	no	no	no	yes

5 CONCLUSION

Deformation analyses by the Caspary approach is the seventh method described an implemented in the same simulated geodetic network. This methodology is developed by W. F. Caspary, and it is presented on his monograph and in other literature (Caspary, 1988; Mihailović and Aleksić, 1994). Caspary method has many similarities with the method of Hannover, but it includes additionally geological and geophysical analysis to define which points need to be treated as conditionally stable (Mihailović and Aleksić, 1994). These points are analysed additionally for their stability by the explained methodology, moved points in the further analyses will belong in the same group with the object points.

Stable points need to define the geodetic datum in both epochs. To transform the geodetic datum of the first and second epoch into stable points was used S-transformation. After the datum transformations, coordinate differences between the second and first epoch are defining point's displacements. Caspary approach foresees to show displacements also graphically, unstable points from reference block are confirmed by error ellipses additionally while the displacement vector needs to be out of the error ellipse which origin is defined from coordinates of the first epoch.

Results presented by Caspary method have minor differences with results obtained by other methods of deformation analyses published in the previous articles of one of the authors of this article, common points are confirmed as unstable by all methods, and estimated displacements are very close to simulated displacements.

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DEFORMACIJSKA ANALIZA PO POSTOPKU CASPARY

OSNOVNE INFORMACIJE O ČLANKU:

GLEJ STRAN 68

1 UVOD

Postopek Caspary je razvil W. F. Caspary na Univerzi Novi južni Wales v Sydneyju v Avstraliji (Caspary, 1988). V postopku uporabimo geodetske meritve, ki jih izvedemo v dveh neodvisnih terminskih izmerah, vključuje pa tudi geološke in geofizikalne analize (Mihailović in Aleksić, 1994). S temi analizami lahko dobimo podatke o stabilnosti točk, vendar določitev stabilnih točk med dvema terminskima izmerama opravimo s statističnim testiranjem premikov točk. Postopek Caspary je podoben postopku Hannover, razlika med njima je v tem, da se pri prvem na koncu predvideva uporaba transformacije S v datum tistih referenčnih točk, ki jih s predhodno opravljenimi statističnimi testi določimo kot stabilne (Mihailović in Aleksić, 1994).

Postopke, pri katerih se za izračun premikov uporabljajo geodetske meritve, v splošnem delimo na štiri korake. Podobno lahko delimo postopek Caspary (Caspary, 1988; Mihailović in Aleksić, 1994):

1. Najprej vzpostavimo geodetsko mrežo, določimo plan izmere, po njem izvedemo izmero in dobimo meritve zahtevane natančnosti.
2. V drugem koraku lociramo in odstranimo morebitne grobe pogreške med meritvami, izvedemo izravnavo in izračunamo izravnane koordinate točk.
3. V tretjem koraku ugotovimo, katere referenčne točke so se med dvema terminskima izmerama premaknile. V nadaljnjih izračunih teh točk ne smemo uporabiti kot datumske točke.
4. V zadnjem koraku naredimo transformacijo S ter grafično prikažemo premike točk na objektu in tistih referenčnih točk, za katere smo ugotovili, da so se premaknile.

2 TEORETIČNO OZADJE

2.1 Vzpostavitev geodetske mreže in določitev plana izmere

Geometrija geodetske mreže je odvisna od konfiguracije terena ter vrste in velikosti objekta, ki ga želimo spremljati. V danih terenskih razmerah moramo za spremljanje stabilnosti objekta vzpostaviti takšno geodetsko mrežo, da bomo dosegli projektirano natančnost meritev (Mihailović in Aleksić, 2008). Plan meritev moramo sestaviti tako, da ga bo ob upoštevanju konfiguracije terena mogoče realizirati. Za določitev geometrije geodetske mreže in plana izmere uporabimo optimizacijo prvega reda, za določitev uteži načrtovanih meritev pa optimizacijo drugega reda.

2.2 Izravnava meritev in ugotovitev morebitno grobo pogrešenih meritev

Postopek Caspary je razvrščen v skupino metod, s katerimi analiziramo meritve dveh terminskih izmer. Koordinatne razlike identičnih točk med dvema terminskima izmerama predstavimo kot premike. Najprej moramo testirati homogenost natančnosti meritev, torej če sta a-posteriori referenčni varianci $\hat{\sigma}_{0i}^2$ obeh terminskih izmer statistično enaki. Test opravimo s preizkusom ničelne H_0 hipoteze (Ambrožič, 2001; Caspary, 1988; Mihailović in Aleksić, 1994):

$$H_0 : E\left(\hat{\sigma}_{01}^2\right) = (\hat{\sigma}_{02}^2) = \sigma_0^2, \quad (1)$$

$$H_a : E\left(\hat{\sigma}_{01}^2\right) \neq (\hat{\sigma}_{02}^2) \neq \sigma_0^2. \quad (2)$$

Preverjanje ničelne hipoteze opravimo s testno statistiko T, ki se porazdeljuje po porazdelitvi F:

$$T = \frac{\hat{\sigma}_{01}^2}{\hat{\sigma}_{02}^2} \leq F_{1-\alpha, f_1, f_2}, \quad (3)$$

$$f_1 = n_1 - u_1 + d_1, \quad (4)$$

$$f_2 = n_2 - u_2 + d_2, \quad (5)$$

α – izbrana stopnja značilnosti testa,

f_i – število nadštevilnih meritev (prostostnih stopenj) v posamezni terminski izmeri,

n_i – število meritev v posamezni terminski izmeri,

u_i – število neznank v posamezni terminski izmeri,

d_i – defekt datuma (defekt ranga matrike normalnih enačb) v posamezni terminski izmeri.

Če ničelne hipoteze ne moremo zavrniti, izračunamo skupno a-posteriori referenčno varianco (Caspary, 1988; Mihailović in Aleksić, 1994):

$$s^2 = \frac{f_1\hat{\sigma}_{01}^2 + f_2\hat{\sigma}_{02}^2}{f_1 + f_2} = \frac{q}{f}. \quad (6)$$

Postopek Caspary predvideva, da je datum geodetske mreže določen z notranjimi vezmi, zato morajo biti v izravnavi izpolnjeni naslednji pogoji (Kuang, 1996; Caspary, 1988):

$$\mathbf{v}_i^{\mathrm{T}}\mathbf{P}_i\mathbf{v}_i = min., \quad (7)$$

$$\mathbf{x}_i^{\mathrm{T}}\mathbf{x}_i = min. \text{ in } \hat{\mathbf{x}}_i = \mathbf{x}_{0i} + \mathbf{x}_i, \quad (8)$$

$\mathbf{v}_i$ – vektor popravkov meritev v posamezni terminski izmeri,

$\mathbf{P}_i$ – matrika uteži v posamezni terminski izmeri,

$\mathbf{x}_{0i}$ – vektor približnih vrednosti koordinat v posamezni terminski izmeri,

$\mathbf{x}_i$ – vektor popravkov približnih vrednosti koordinat v posamezni terminski izmeri,

$\hat{\mathbf{x}}_i$ – vektor izravnanih koordinat v posamezni terminski izmeri.

Geodetsko mrežo v posamezni terminski izmeri izravnamo kot prosto mrežo, orientacijske neznanke (in morebitno neznanko merila mreže) moramo odstraniti z Gaußovo metodo eliminacije ali drugimi metodami (Mihailović in Aleksić, 1994; Mihailović, 1981). Za odkrivanje grobo pogrešenih meritev lahko uporabimo Baardovo metodo Data Snooping, Popeovo metodo Tau ali dansko metodo (Caspary, 1988; Grigillo in Stopar, 2003). Pred izravnavo moramo uskladiti tudi natančnosti kotnih in dolžinskih meritev (Ambrožič, 2004).

2.3 Testiranje nestabilnih točk po postopku Caspary

Postopek Caspary je, v postopku določitve stabilnih referenčnih točk, podoben nekaterim drugim metodam deformacijske analize, še posebej postopku Hannover (Mihailović in Aleksić, 1994). Po izravnavi geodetske mreže kot proste mreže sta vektor izravnanih koordinatnih razlik Δ in psevdoinverzna matrika kofaktorjev izravnanih koordinatnih razlik $\mathbf{Q}_{\Delta\Delta}^{-}$ definirana kot (Caspary, 1988):

$$\Delta = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1, \tag{9}$$

$$\mathbf{Q}_{\Delta\Delta}^{-} = \mathbf{Q}_{\hat{x}\hat{x}1}^{+} + \mathbf{Q}_{\hat{x}\hat{x}2}^{+} = \mathbf{B}_1^{\mathrm{T}}\mathbf{P}_1\mathbf{B}_1 + \mathbf{B}_2^{\mathrm{T}}\mathbf{P}_2\mathbf{B}_2, \tag{10}$$

$\mathbf{B}_i$ – matrika koeficientov enačb popravkov meritev v posamezni terminski izmeri.

Vektor Δ (9) moramo razstaviti na podvektorja $\Delta^{\mathrm{T}} = (\Delta_n^{\mathrm{T}}\ \Delta_p^{\mathrm{T}})$. V podvektorju Δ_n^{T} so koordinatne razlike referenčnih točk, ki jih bomo v nadaljevanju testirali glede stabilnosti, v podvektorju Δ_p^{T} pa so koordinatne razlike točk na objektu, ki jih bomo v nadaljnjih korakih obravnavali kot nestabilne točke:

$$\Delta = \begin{bmatrix} \Delta_n \\ \Delta_p \end{bmatrix}. \tag{11}$$

Podobno razstavimo tudi pripadajočo matriko $\mathbf{Q}_{\Delta\Delta}^{-}$ (Caspary, 1988):

$$\mathbf{Q}_{\Delta\Delta}^{-} = \mathbf{P}_{\Delta\Delta} = \begin{bmatrix} \mathbf{P}_{nn} & \mathbf{P}_{np} \\ \mathbf{P}_{pn} & \mathbf{P}_{pp} \end{bmatrix}. \tag{12}$$

Stabilnost referenčnih točk med dvema terminskima izmerama določimo s testiranjem naslednje hipoteze (Caspary, 1988):

$$H_0 : E\left(\hat{\mathbf{x}}_{n1}\right) = \left(\hat{\mathbf{x}}_{n2}\right) = \hat{\mathbf{x}}_n, \tag{13}$$

$$H_a : E\left(\hat{\mathbf{x}}_{n1}\right) \neq \left(\hat{\mathbf{x}}_{n2}\right) \neq \hat{\mathbf{x}}_n. \tag{14}$$

Če ničelne hipoteze ne moremo zavrniti, pomeni, da so vse v testiranje vključene referenčne točke v dveh terminskih izmerah ostale stabilne, v nasprotnem primeru je med dvema terminskima izmerama nekaj referenčnih točk spremenilo svoje koordinate. Testiranje ničelne hipoteze (13) opravimo s testno statistiko T, ki se porazdeljuje po porazdelitvi F (Caspary, 1988):

$$T = \frac{\left(q_\Delta\right)_n / f_\Delta}{q / f} = \frac{\left(q_\Delta\right)_n / f_\Delta}{S^2} \sim F_{(1-a, f_\Delta, f)}, \tag{15}$$

kjer je:

$$\left(q_\Delta\right)_n = \Delta_n^{\mathrm{T}}\mathbf{Q}_{nn}^{-}\Delta_n, \tag{16}$$

$$f_\Delta = 2m - d, \tag{17}$$

m – število referenčnih točk, ki so vključene v testiranje.

Matriko $\mathbf{Q}_{nn}^{-}$ izračunamo z naslednjo enačbo (Caspary, 1988):

$$\mathbf{Q}_{nn}^{-} = \mathbf{P}_{nn} - \mathbf{P}_{np}\mathbf{P}_{pp}^{-1}\mathbf{P}_{pn}. \tag{18}$$

Če točke geodetske mreže niso razdeljene na referenčne točke in točke na objektu, torej obravnavamo vse točke mreže kot referenčne, potem je matrika $\mathbf{Q}_{nn}^{-}$ (18) enaka matriki $\mathbf{Q}_{\Delta\Delta}^{-}$ (12). Vrednost testne statistike T izračunano z enačbo (15), primerjamo z ustrezno kritično vrednostjo $F_{(1-a,\, f_\Delta,\, f)}$. Če je $T \leq F_{(1-a,\, f_\Delta,\, f)}$, potem ničelne hipoteze ne moremo zavrniti, če je $T > F_{(1-a,\, f_\Delta,\, f)}$, pa ničelno hipotezo zavrnemo. Zavrnitev ničelne hipoteze pomeni, da imamo med referenčnimi točkami tudi točke, ki so se dvema terminskima izmerama premaknile. Tako moramo opraviti dodatne analize, da odkrijemo nestabilne referenčne točke.

Postopek določitve nestabilnih referenčnih točk izvedemo iterativno. Razstavljanje vektorja Δ in matrike $\mathbf{Q}_{\Delta\Delta}^{-}$ naredimo podobno, kot smo zapisali v enačbah (11) in (12). V posamezni iteraciji so v podvektorju Δ_n koordinatne razlike vseh referenčnih točk razen ene, za katere smo predpostavili, da so stabilne in jih bomo v nadaljevanju testirali, ali so res stabilne. V podvektorju Δ_p sta le koordinatni razliki (za 2D-mrežo) samo ene referenčne točke (tiste, ki je nismo vključili v podvektor Δ_n) (Caspary, 1988). Pripadajočo matriko $\mathbf{Q}_{\Delta\Delta}^{-}$ razstavimo podobno: podmatrika $\mathbf{P}_{nn}$ vsebuje elemente vseh referenčnih točk razen ene, podmatrika $\mathbf{P}_{pp}$ pa elemente le ene točke (tiste, ki je ni v podmatriki $\mathbf{P}_{nn}$). Kvadratno formo $(q_\Delta)_n$, enačba (16), lahko razstavimo na dva dela. To naredimo po transformaciji vektorja Δ na podvektorja naslednje oblike (Caspary, 1988):

$$\begin{bmatrix} \overline{} \\ \overline{} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{P}_{pp}\mathbf{P}_{pn} & \mathbf{I} \end{bmatrix} \Delta \tag{19}$$

Podvektor $\overline{\Delta}_p$ se nanaša na točko, ki jo testiramo, podvektor $\overline{\Delta}_n$ pa se nanaša na vse preostale referenčne točke. Razstavljanje vektorja Δ in matrike $\mathbf{Q}_{\Delta\Delta}^{-}$ ter izračun $(q_\Delta)_p$ naredimo za vsako točko posebej. Iteracij je toliko, kolikor imamo referenčnih točk, za katere smo predpostavili, da so stabilne. Kot nestabilno točko določimo točko, za katero izračunamo po vsaki iteraciji največjo vrednost $(q_\Delta)_p$ (Caspary, 1988):

$$(q_\Delta)_p = \overline{\Delta}_p^{\mathbf{T}} \mathbf{P}_{pp} \overline{\Delta}_p. \tag{20}$$

V naslednji iteraciji prej določeno nestabilno referenčno točko prestavimo v skupino točk na objektu. Razstavljanje vektorja Δ (11) in matrike $\mathbf{Q}_{\Delta\Delta}^{-}$ (12) nadaljujemo iterativno, dokler ne odkrijemo vseh nestabilnih referenčnih točk. Testno statistiko, določeno v (15), ponavljamo tolikokrat, dokler ničelne hipoteze (13) ne moremo zavrniti, kar pomeni, da so vse preostale referenčne točke stabilne.

2.4 Izračun premikov

Ko odkrijemo vse točke, ki niso stabilne, izračunamo premike točk tako, da vektor Δ in pripadajočo matriko $\mathbf{Q}_{\Delta\Delta}^{-}$ razstavimo na (Caspary, 1988):

$$\Delta = \begin{bmatrix} \Delta_r \\ \Delta_o \end{bmatrix}, \tag{21}$$

$$\mathbf{Q}_{\Delta\Delta}^{-} = \mathbf{P}_{\Delta\Delta} = \begin{bmatrix} \mathbf{P}_{rr} & \mathbf{P}_{ro} \\ \mathbf{P}_{or} & \mathbf{P}_{oo} \end{bmatrix}. \tag{22}$$

V podvektorju Δ_r so koordinatne razlike stabilnih točk, medtem ko so v podvektorju Δ_o koordinatne razlike točk, ki so se premaknile. Za razstavitev matrike $\mathbf{Q}_{\Delta\Delta}^{-}$ velja podobno: elementi podmatrike $\mathbf{P}_{rr}$ se nanašajo na stabilne točke, medtem ko se elementi podmatrike $\mathbf{P}_{oo}$ nanašajo na točke, ki so se premaknile, torej gre za točke na objektu in nestabilne referenčne točke. V zadnji transformaciji, ki jo izvedemo, bodo točke, ki so stabilne, določale nov datum geodetske mreže. Za transformacijo geodetske mreže uporabimo transformacijo S za vsako terminsko izmero posebej, datum transformirane mreže je določen s stabilnimi točkami. Vektor Δ in matriko $\mathbf{Q}_{\Delta\Delta}$ transformiramo v nov geodetski datum stabilnih točk z naslednjimi enačbami (Stopar in Marjetič, 2007):

$$\Delta_{stab} = \mathbf{S}_{stab}\Delta, \tag{23}$$

$$\mathbf{Q}_{\Delta\Delta stab} = \mathbf{S}_{stab}\mathbf{Q}_{\Delta\Delta}\mathbf{S}_{stab}^{\mathrm{T}}, \tag{24}$$

$$\mathbf{S}_{stab} = \mathbf{I} - \mathbf{H}(\mathbf{H}\mathbf{E}_{stab}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{E}_{stab}. \tag{25}$$

V matriki $\mathbf{E}_{stab}$ so vsi elementi enaki nič, le diagonalni elementi imajo vrednosti enake ena na mestih, ki pripadajo koordinatni komponenti stabilne referenčne točke. Matrika **I** je enotska matrika, matrika **H** je matrika geodetskega datuma, določenega z notranjimi vezmi (Krüger, 1980). Premike točk na objektu in nestabilnih referenčnih točk izračunamo z naslednjo enačbo (Caspary, 1988):

$$\overline{\Delta}_o = \mathbf{P}_{oo}^{-1}\mathbf{P}_{or}\Delta_r + \Delta_o. \tag{26}$$

Postopek Caspary predvideva tudi grafično predstavitev vektorjev premikov in elips zaupanja. Elipse zaupanja koordinat točk so določene s stopnjo zaupanja 95 % (Caspary, 1988). V grafični predstavitvi določajo koordinate točk prve (ničelne) terminske izmere središča elips zaupanja, razlike koordinat med terminskima izmerama pa določajo dolžino vektorjev premikov točk. Če se konec vektorja premika nahaja izven elipse zaupanja, lahko ponovno potrdimo, da se je točka premaknila, če je konec vektorja premika znotraj elipse zaupanja, pa lahko trdimo, da se točka ni premaknila. Iz elementov matrike $\mathbf{Q}_{\Delta\Delta stab} = \mathbf{Q}_{\Delta\Delta stab1} + \mathbf{Q}_{\Delta\Delta stab2}$ izračunamo elemente elips zaupanja na naslednji način (Caspary, 1988):

$$a = s\sqrt{\lambda_1 2F_{1-\alpha,2,f}}, \tag{27}$$

$$b = s\sqrt{\lambda_2 2F_{1-\alpha,2,f}}, \tag{28}$$

$$\tan 2\Theta = \frac{2q_{\hat{x}\hat{y}}}{q_{\hat{x}\hat{x}} - q_{\hat{y}\hat{y}}}, \tag{29}$$

$$\lambda_1 = \frac{1}{2}\left(q_{\hat{x}\hat{x}} + q_{\hat{y}\hat{y}} + z\right), \tag{30}$$

$$\lambda_2 = \frac{1}{2}\left(q_{\hat{x}\hat{x}} + q_{\hat{y}\hat{y}} - z\right), \tag{31}$$

$$z^2 = \left(q_{\hat{x}\hat{x}} - q_{\hat{y}\hat{y}}\right)^2 + 4q_{\hat{x}\hat{y}}q_{\hat{x}\hat{y}}. \tag{32}$$

Grafična predstavitev dobljenih rezultatov je zadnji korak pristopa Caspary in jo izvedemo po izračunu elementov elips zaupanja, predstavljenih z enačbami od (27) do (32).

3 RAČUNSKI PRIMER

Uporabnost postopka Caspary želimo testirati na simulirani geodetski mreži, sestavljeni iz 7 točk ter merjenih 24 horizontalnih smeri in 24 dolžin. Za vrednost a-priori variance za smeri izberemo σ_{si} = 1", za vrednost a-priori variance za dolžine pa σ_{di} = 5 mm. Za obe terminski izmeri je plan meritev enak. Isto geodetsko mrežo so uporabili tudi drugi raziskovalci, ko so testirali druge postopke deformacijske analize:

- Hannover (Ambrožič, 2001),
- Karlsruhe (Ambrožič, 2004),
- Delft (Marjetič, Zemljak in Ambrožič, 2013),
- Fredericton (Vrečko in Ambrožič, 2013),
- München (Soldo in Ambrožič, 2018),
- robustne metode (Ambrožič et al., 2019).

Postopek Caspary predvideva, da so v izravnavi mreže izpolnjeni pogoji, določeni v enačbah (7) in (8). Matriko kofaktorjev neznank izračunamo z naslednjim izrazom (Krüger, 1980):

$$\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = (\mathbf{B}^{\mathrm{T}}\mathbf{P}\mathbf{B} + \mathbf{H}\mathbf{H}^{\mathrm{T}})^{-1} - \mathbf{H}\mathbf{H}^{\mathrm{T}}. \tag{33}$$

Datumska matrika $\mathbf{H}^{\mathrm{T}}$ je odvisna od vrste izvedenih meritev v geodetski mreži in je določena glede na defekt datuma geodetske mreže, ki je v našem primeru v obeh terminskih izmerah enak tri (Krüger, 1980). Vsako terminsko izmero smo izravnali, rezultate prikazujemo v preglednici 1.

Preglednica 1: Rezultati izravnave.

	Prva terminska izmera	Druga terminska izmera
	$i = 1$	$i = 2$
$\hat{\sigma}_{0i}^2$	0,941	1,337
n_i	48	48
u_i	14 + 7	14 + 7
d_i	3	3
f_i	30	30
s^2; enačba (6)	1,337	
f; enačba (6)	60	

Iz rezultatov, predstavljenih v preglednici 1, ugotovimo, da smo dosegli homogeno natančnost meritev v obeh terminskih izmerah. Vrednost skupne a-posteriori referenčne variance s^2 uporabimo v nadaljnjih izračunih. Vektor Δ in matriko $\mathbf{Q}_{\Delta\Delta}^{-}$ izračunamo po enačbah (9) in (10). Ker geodetske mreže nismo razdelili na referenčne točke in točke na objektu, sta matriki $\mathbf{Q}_{nn}^{-}$ in $\mathbf{P}_{nn}$ enaki. V naslednjem koraku testiramo stabilnost točk med dvema terminskima izmerama z ničelno hipotezo (13). Izračunamo vrednost testne statistike po enačbi (15) in ugotovimo, da moramo ničelno hipotezo (13) zavrniti. Rezultate tega izračuna predstavljamo v preglednici 2.

Preglednica 2: Rezultati testiranja skladnosti.

Parameter	m	f_Δ	$(q_\Delta)_n$	T	$F_{(0.975,11,60)}$
Vrednost	7	11	1772,21	141,48	1,95

Test skladnosti je potrdil, da so se nekatere točke geodetske mreže med dvema terminskima izmerama premaknile. Zato postopek nadaljujemo z izračuni, opisanimi v podpoglavju 2.3. Rezultate teh izračunov prikazujemo v preglednici 3, rezultate testne statistike (15) in kritične vrednosti po vsaki iteraciji pa v preglednici 4.

Preglednica 3: Določitev nestabilnih referenčnih točk.

Točka	1. iteracija	2. iteracija	3. iteracija	4. iteracija
	$(q_\Delta)_p$	$(q_\Delta)_p$	$(q_\Delta)_p$	$(q_\Delta)_p$
1	**755,51**			
2	562,59	321,26	**505,70**	
3	415,29	347,99	394,94	**146,15**
4	94,53	99,03	52,00	75,72
5	67,62	75,56	17,13	3,87
6	8,99	95,81	52,00	0,51
7	664,76	**363,61**		

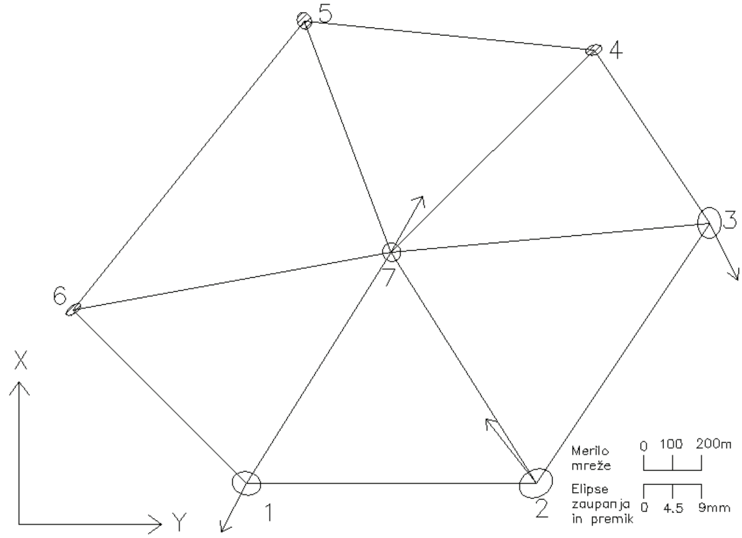
Preglednica 4: Rezultati testne statistike (15) in kritične vrednosti.

Parameter	1. iteracija	2. iteracija	3. iteracija	4. iteracija	5. iteracija
m	7	6	5	4	3
f_Δ	11	9	7	5	3
T	141,48	99,20	81,93	25,89	0,36
F	1,95	2,04	2,17	2,37	2,76

Iz predstavljenih rezultatov sklepamo, da so se točke 1, 2, 3 in 7 premaknile, točke 4, 5 in 6 pa so stabilne. V peti iteraciji ugotovimo, da s testom skladnosti ne moremo zavrniti ničelne hipoteze (13). V naslednjem koraku potrjeno stabilne točke uporabimo kot točke, ki definirajo datum mreže v transformaciji S, ki je predstavljena z enačbami (23), (24) in (25). Po transformaciji S, s katero v obeh terminskih izmerah zagotovimo identični datum geodetske mreže, izračunamo premike točk in elemente elips zaupanja, ki jih prikazujemo v preglednici 5. Elipse zaupanja in ugotovljene vektorje premikov grafično prikazujemo na sliki 1.

Preglednica 5: Premiki in elementi elips zaupanja.

Točka	Premiki		Elipse zaupanja		
	d_y [mm]	d_x [mm]	a [mm]	b [mm]	Θ [°]
1	– 19,2	– 37,9	11,1	9,2	104
2	– 38,4	49,4	13,2	10,4	62
3	20,8	– 43,9	11,9	9,1	2
4	–	–	6,4	3,7	74
5	–	–	6,2	5,4	159
6	–	–	6,5	2,9	55
7	23,9	43,1	7,2	6,7	151



Slika 1: Elipse zaupanja in premiki točk.

4 PRIMERJAVA Z REZULTATI DRUGIH POSTOPKOV

Primerjavo rezultatov postopka Caspary in rezultatov postopkov Hannover, Karlsruhe, Delft, Fredericton in München predstavljamo v preglednici 6. Z uporabo vseh postopkov trdimo, da so točke 4, 5 in 6 stabilne, točke 1, 2, 3 in 7 pa nestabilne. Opazimo lahko le majhno razliko v rezultatih izračunanih premikov, vendar so izračunane vrednosti premikov zelo podobne simuliranim premikom.

Preglednica 6: Simulirani premiki točk mreže in rezultati deformacijske analize po postopkih Hannover, Karlsruhe, Delft, Fredericton, München in Caspary.

Točka		**1**	**2**	**3**	**4**	**5**	**6**	**7**
Simulirano	d_y [mm]	– 20,0	– 30,0	25,0	0,0	0,0	0,0	25,0
	d_x [mm]	– 34,6	52,0	– 43,3	0,0	0,0	0,0	43,3
	d [mm]	40,0	60,0	50,0	0,0	0,0	0,0	50,0
	υ [°]	210	330	150	–	–	–	30
Hannover	d_y [mm]	– 19,6	– 38,7	20,6	– 4,0	– 6,4	3,3	23,6
	d_x [mm]	– 38,0	49,0	– 44,3	5,1	– 7,1	– 10,6	42,9
	d [mm]	42,8	62,4	48,9	6,5	10,0	11,1	49,0
	υ [°]	207	322	155	322	222	163	29
	Premik	da	da	da	ne	ne	ne	da
Karlsruhe	d_y [mm]	– 19,7	– 38,8	20,6	–	–	–	23,6
	d_x [mm]	– 38,0	49,0	– 44,4	–	–	–	42,9
	d [mm]	42,8	62,5	48,9	–	–	–	49,0
	υ [°]	207	322	155	–	–	–	29
	Premik	da	da	da	ne	ne	ne	da

Točka		1	2	3	4	5	6	7
Delft	d_y [mm]	– 19,4	– 38,1	21,4	0,7	– 0,8	0,0	24,0
	d_x [mm]	– 37,5	49,5	– 43,5	1,0	– 2,3	1,3	42,9
	d [mm]	42,2	62,5	48,5	1,2	2,4	1,3	49,2
	υ [°]	207	322	154	35	199	0	29
	Premik	da	da	da	ne	ne	ne	da
Fredericton	d_y [mm]	– 19,6	– 38,7	20,6	–	–	–	23,6
	d_x [mm]	– 38,0	49,0	– 44,3	–	–	–	42,9
	d [mm]	42,8	62,5	48,9	–	–	–	48,9
	υ [°]	207	322	155	–	–	–	29
	Premik	da	da	da	ne	ne	ne	da
München	d_y [mm]	– 19,5	– 38,2	21,4	0,7	– 0,8	0,0	24,0
	d_x [mm]	– 37,6	49,5	– 43,6	1,0	– 2,2	1,4	42,9
	d [mm]	42,4	62,5	48,6	1,2	2,3	1,4	49,2
	υ [°]	207	322	154	35	200	0	29
	Premik	da	da	da	ne	ne	ne	da
Caspary	d_y [mm]	– 19,2	– 38,4	20,8	–	–	–	23,9
	d_x [mm]	– 37,9	49,4	– 43,9	–	–	–	43,1
	d [mm]	42,5	62,5	48,6	–	–	–	49,2
	υ [°]	207	322	154	–	–	–	9
	Premik	da	da	da	ne	ne	ne	da

5 SKLEP

Deformacijska analiza po postopku Caspary je sedma opisana metoda, izvedena na isti simulirani geodetski mreži. Postopek je razvil W. F. Caspary, predstavil ga je v monografiji (Caspary, 1988), obravnavali pa so ga tudi drugi avtorji (Mihailović in Aleksić, 1994). Postopek Caspary je delno podoben postopku Hannover, vključuje pa dodatne geološke in geofizikalne analize, ki jih uporabimo za odločitev, katere referenčne točke lahko obravnavamo kot pogojno stabilne (Mihailović in Aleksić, 1994). Stabilnost teh točk dodatno analiziramo s predstavljeno metodologijo. Referenčne točke, za katere smo ugotovili, da niso stabilne, pa v nadaljnjih analizah obravnavamo kot točke na objektu.

Stabilne referenčne točke morajo določati geodetski datum v obeh terminskih izmerah. Za transformacijo geodetske mreže prve in druge terminske izmere v datum stabilnih referenčnih točk uporabimo transformacijo S. Po transformaciji mreže prve in druge terminske izmere v geodetski datum stabilnih referenčnih točk izračunamo koordinatne razlike točk med drugo in prvo terminsko izmero. Postopek Caspary predvideva grafično predstavitev vektorjev premikov in elips zaupanja. Vektor premika nestabilnih referenčnih točk, katerega začetek določajo koordinate točke iz prve terminske izmere in ima konec izven elipse zaupanja, potrdi, da se je referenčna točka premaknila.

Rezultati, ki jih dobimo s postopkom Caspary, se le malo razlikujejo od rezultatov, ki jih dobimo z drugimi postopki deformacijske analize in so objavljeni v predhodnih člankih enega od avtorjev tega članka. Z vsemi pristopi deformacijske analize smo potrdili iste nestabilne točke, izračunani premiki pa so zelo podobni simuliranim premikom.

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Veton Hamza, mag. inž. geod. in geoinf.
Univerza v Ljubljani, Fakulteta za gradbeništvo in geodezijo
Jamova cesta 2, SI-1000 Ljubljana
e-naslov: veton.hamza@fgg.uni-lj.si

prof. dr. Bojan Stopar, univ. dipl. inž. geod.
Univerza v Ljubljani, Fakulteta za gradbeništvo in geodezijo
Jamova cesta 2, SI-1000 Ljubljana
e-naslov: bojan.stopar@fgg.uni-lj.si

izr. prof. dr. Tomaž Ambrozič, univ. dipl. inž. geod., univ.dipl. inž. rud.
Univerza v Ljubljani, Fakulteta za gradbeništvo in geodezijo
Jamova cesta 2, SI-1000 Ljubljana
e-naslov: tomaz.ambrozic@fgg.uni-lj.si