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OCENA TOČNOSTI IN PRIMERJAVA METOD INTERPOLACIJ MODELOV GEOIDA

ACCURACY ASSESSMENT AND COMPARISON OF INTERPOLATION METHODS ON GEOID MODELS

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IZVLEČEK

V članku obravnavamo točnost šestih prostorskih interpolacijskih metod na mrežnih modelih geoida, in sicer metodo inverzne utežene razdalje, kriging, metodo najmanjše ukrivljenosti, naravnih sosedov, radialno bazno funkcijo ter metodo drsečega povprečja. Mrežni modeli so bili določeni na podlagi globalnih geopotencialnih modelov EGM2008, EIGEN-6C4 in GECO. S programsko rešitvijo GoldenSoftware Surfer 13 *smo v ta namen izdelali 18 modelov geoida, pri čemer smo uporabili različne kombinacije šestih interpolacijskih metod z navedenimi geopotencialnimi modeli. Pri tem smo obravnavali območje Hrvaške z deli sosednjih držav. Med celotnim procesom smo ohranili enake vrednosti interpolacijskih parametrov. Modele smo razvili tako, da smo lahko modelirano vrednost geoidne ondulacije odšteli od ondulacije neodvisnega modela v isti točki. Tako pridobljene razlike so bile uporabljene za oceno kakovosti interpoliranih modelov geoida. Na podlagi primerjalne analize kazalnikov kakovosti v obliki statističnih in grafičnih kazalnikov je mogoče ugotoviti, da so najboljši rezultati pri uporabi radialne bazne funkcije, sledi interpolacijska metoda kriging in nato metoda naravnih sosedov.*

ABSTRACT

In this paper the accuracy of 6 spatial interpolation methods on geoid grid models are tested and compared: Inverse Distance Weighting, Kriging, Minimum Curvature, Natural Neighbour, Radial Basis Function and Moving Average. Grid models are derived from three global geopotential models: EGM2008, EIGEN-6C4 and GECO. Totally 18 geoid models in forms of equiangular grid models are created in GoldenSoftware Surfer 13 *for this purpose, by combining 6 methods of interpolation and 3 global models, whose size and position cover a wide area of Croatia with parts of neighbouring countries. Default interpolation parameters were kept during the process. Models were created in a way that modelled values of geoid undulations can be subtracted from undulations of independent models in equal points. Residuals calculated in this way served as a base for quality assessment. Comparative analysis of quality indicators, statistics and graphical indicators of residuals led to a conclusion that the best-fitted methods of interpolation for this specific case are Radial Basis Function, Kriging, and Natural Neighbour, in that order.*

KLJUČNE BESEDE

geoid, model geoida, prostorske interpolacije, globalni geopotencialni model, interpolacije, Hrvaška

KEY WORDS

geoid, geoid model, spatial interpolation, global geopotential model, interpolation, Croatia

1 INTRODUCTION

Recently we are witnessing great progress that is being made in the quality and resolution of global geopotential models. The creation and development of high-quality global geopotential models, as mathematical functions that approximate gravity field of the Earth in 3D space, is very important in numerous aspects. Namely, these models can be considered as a reference for geodesy itself (Barthelmes, 2014). Among other things, they also serve for determination of absolute Dynamic Ocean Topography in combination with altimetry observations in ocean areas, as well as provide the possibility of calculation of orthometric heights and height differences in combination with GNSS positioning without the need for levelling (Pavlis et al., 2012). There is also a possibility that, in the future, a unique global geoid model obtained from a high-accuracy and high-resolution global geopotential model could serve as a reference surface for creation and realization of a global vertical datum (Balasubramania, 1994.; Schwarz et al., 1987; Ihde and Sanchez, 2005). But global geopotential models also carry a lot of vital information about Earth and its inner and outer composition, so they are not relevant only for geodesy but for all of the geosciences. Consequently, geodesy is faced with an important task of providing global geopotential model and all of its products, which are of highest possible quality, which have known accuracy and which are easily accessible (Barthelmes, 2013).

The gravity acceleration or just gravity acting on a unit mass that is fixed on and rotates with Earth consists of centrifugal acceleration, which arises as a result of Earth's rotation, and gravitation, which arises from the attraction between Earth and unit mass according to Newton's law of gravitation (Torge, 1989). The gravity potential of the earth W, which is connected to gravity by relation $\vec{g} = gradW$, acting on a unit mass is a sum of gravitational potential V and centrifugal potential Φ (Hofmann-Wellenhof and Moritz, 2006):

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$$W = V + \Phi = G\iiint \frac{\rho}{l} dv + \frac{\omega^2}{2} p^2, \quad (1)$$

where G is the gravitational constant, ω is the angular velocity of Earth's rotation, p is normal to Earth's rotation axis, ρ is the volume density and dv is the volume element.

Modelling of the centrifugal potential is rather simple, due to the facts that angular velocity of Earth's rotation is known with a rather high accuracy (Barthelmes, 2014) and it being a simple analytic function. The main problem lies in the modelling of the gravitational potential because the density of the Earth is generally not known. If by some chance function of Earth's density were known, gravity potential at any point could be computed simply as a function of point's position. As this is obviously not the case, we are forced to approximate or model the Earth's gravity field using different gravity field observations (Torge, 2001).

There are numerous ways to mathematically describe or represent gravity potential, but the most commonly used method in practice are spherical harmonics, which can be used to represent any harmonic potential. As gravitational potential V is a harmonic function outside of the Earth's surface, it can be expressed at any point (r, λ, φ) on and above Earth's surface by summing up over degree and order of a spherical harmonic expansion (Barthelmes, 2014):

$$V(r,\lambda,\varphi) = \frac{GM}{r} \sum_{l=0}^{l_{\max}} \sum_{m=0}^{l} \left(\frac{R}{r}\right)^l P_{lm}(\sin\varphi)\left(C_{lm}\cos m\lambda + S_{lm}\sin m\lambda\right), \quad (2)$$

where (r, λ, φ) are spherical geocentric coordinates of computation point, R is a reference radius, M is a mass of the Earth, l, m are degree and order of spherical harmonic, P_{lm} are normalized Legendre's functions and C_{m}, S_{lm} are normalized Stokes' coefficients.

One of the upsides of using spherical harmonics to represent gravitational potential is that all other functionals of the gravity field, like the geoid, gravity disturbance, gravity anomaly or height anomaly, can be derived rather simply for all points on or above Earth's surface. The accuracy of the model is mainly affected by the accuracy of Stokes' coefficients and spatial resolution of the model, which depends on upper degree limit of summation $l_{\max}$ (Barthelmes, 2014). Determination of the coefficients of a series expansion can be done by evaluating any observable functionals of gravitational potential V on any point on or above Earth's surface, while the resolution mainly depends on a number of observations and their distance from the centre of Earth's masses r (Torge, 2001).

Present-day global geopotential models are mainly based on observations of Earth's artificial satellites, and are usually divided into satellite-only models, which are modelled solely from satellite data, and combined models, which combine satellite data with terrestrial observation data over the continental areas and altimetry data over the ocean areas (Rapp and Pavlis, 1990; Rapp, 1998). The accuracy and resolution of global geopotential models is increasing steadily over the last 20 years (Koneshov et al., 2013), which is to a large extent connected with the launching of three satellite missions launched partially for this purpose: CHAMP, GRACE and GOCE which implemented many contemporary observation techniques used for the first time in history (Hećimović and Bašić, 2005a, 2005b, 2005c; Touboul et al., 2012).

One of the products of global geopotential models is the geoid, which is usually distributed in the form of a grid model. Representation of a continuous geoid surface from a grid model and prediction of geoid undulations in unknown areas is enabled by use of different spatial interpolation methods. Today, it is normal for a surface modelling software to include a dozen or more different spatial interpolation algorithms for a user to choose from. That is why assessment and comparison of those methods' accuracy are important for geosciences (Gumus and Sen, 2013; Grgić et al., 2015). The aim of this paper is a determination of spatial interpolation accuracy for several different commonly used methods, applied on different grid models of the geoid over a wide area of Croatia.

2 MODELLING

Three publicly available global geopotential models described by spherical harmonic expansion up to the order of 2190 were used in this paper, ordered by year of creation: EGM2008 (Pavlis et al., 2008), EIGEN-6C4 (Förste et al., 2014) and GECO (Gilardoni et al., 2016). All of those models are combined models and among the highest accuracy models available at present. Their spatial resolution is about 10 km, but in practice, there are large areas with scarce terrestrial data so this resolution should be considered true only for the areas with numerous amount of high accuracy observations (Barthelmes, 2014).

EGM2008 is complete to degree and order 2159 and contains additional spherical harmonic coefficients extending up to degree 2190 and order 2159 (Kotsakis and Katsambalos, 2010). EGM2008 is a first model of this kind to be expanded to such a high degree and order, so it confers it is also unprecedented in the level of spatial resolution. Main satellite data used is from GRACE satellite mission (Pavlis et al., 2012). EGM2008 was implemented in the creation of national geoid surface of the Republic of Croatia called HRG2009 (Bašić, 2009; Bašić and Bjelotomić, 2014). EIGEN-6C4 is a first global geopotential model complete to degree and order of 2190. LAGEOS, GRACE and GOCE satellite missions data were used for its computation. Global geopotential model GECO was made by incorporating GOCE mission data into EGM2008 (Yilmaz et al., 2016).

Two sets of three grid models of geoid undulations derived from geopotential models EGM2008, EIGEN-6C4 and GECO were taken over from the web service (Barthelmes and Köhler, 2016). Grid models contain nodes with (λ, φ, N) coordinates with regard to the GRS80 reference ellipsoid (Moritz 1980), where *N* is undulation of the geoid. Grid models cover a wide area of Croatia with parts of neighbouring countries. Grid geometry for the first set of three grid models is presented in Table 1.

Table 1: Geometry of the grid for the first set of models.

Size	97 row x 145 column lines
Number of nodes	14 065
SW corner node	$\lambda = 10.0000°$, $\varphi = 40.0000°$
NE corner node	$\lambda = 22.0000°$, $\varphi = 48.0000°$
Cell dimensions	$\Delta\lambda = 0.0833°$ (5′) ≈ 6.7 km, $\Delta\varphi = 0.0833°$ (5′) ≈ 9.3 km

The geometry of the second set of three grid models is made by moving the grid by the value of 0.0417° (2.5′), which is equal to half of the size of the cell, in directions of both λ and φ. This results with nodes of the second set of grid models being in the exact middle of cells from the first set of grid models. Grid geometry of the second set is shown in Table 2. It can be noticed by comparing the Tables 1 and 2 that the number of columns and rows is lowered by 1 in the second set of grid models. This means that grids of the second set are completely contained within the grids of the first set.

Table 2: Geometry of the grid for the second set of models.

Size	96 row x 144 column lines
Number of nodes	13 824
SW corner node	$\lambda = 10.0417°$, $\varphi = 40.0417°$
NE corner node	$\lambda = 21.9583°$, $\varphi = 47.9583°$
Cell dimensions	$\Delta\lambda = 0.0833°$ (5′) ≈ 6.7 km, $\Delta\varphi = 0.0833°$ (5′) ≈ 9.3 km

This kind of arrangement allows the comparison of modelled values of geoid undulations with so-called real values. One can create new grid models with geometry equal to the second set of models (Table 2) by using different methods of spatial interpolation on the data contained in the first set of models. These grid models, created using different methods of interpolation, will have nodes that planarly correspond with nodes from the second set of models. Modeled undulations of the geoid $\bar{N}$ can then be compared to values of *N* from the second set of models, which are derived directly from the geopotential models and can, in comparison to $\bar{N}$, be considered real or quasi-real values.

This somewhat specific kind of gridding should be highlighted. Gridding, or creation of a grid model, is by definition the creation of a regular equiangular set of data from observation data that is more or less irregularly spatially distributed (Smith and Wessel 1990). However, in this paper gridding is done on data that is also in a form of a grid. This fact shouldn't be considered as a problem but a curiosity, as modelling methods or methods of spatial interpolation aren't exclusively intended to apply on irregular sets of data. Also, modelled values are directly compared with so-called real values, so the conclusions can be safely made based on the differences between those two.

Modelling of geoid undulations was done using *GoldenSoftware Surfer 13* and following 6 methods of spatial interpolation: *Inverse Distance Weighted* (*IDW*), *Kriging*, *Minimum Curvature* (*MC*), *Natural Neighbour* (*NN*), *Radial Basis Function* (*RBF*) and *Moving Average* (*MA*). Default interpolation parameters suggested by *Surfer* were kept in all cases and are presented in Table 3. 18 grid models were created in total, 6 per each of the 3 global geopotential models (EGM2008, EIGEN-6C4 and GECO). Figure 1 shows 6 of those models related to GECO, as a most up-to-date global model included in this paper.

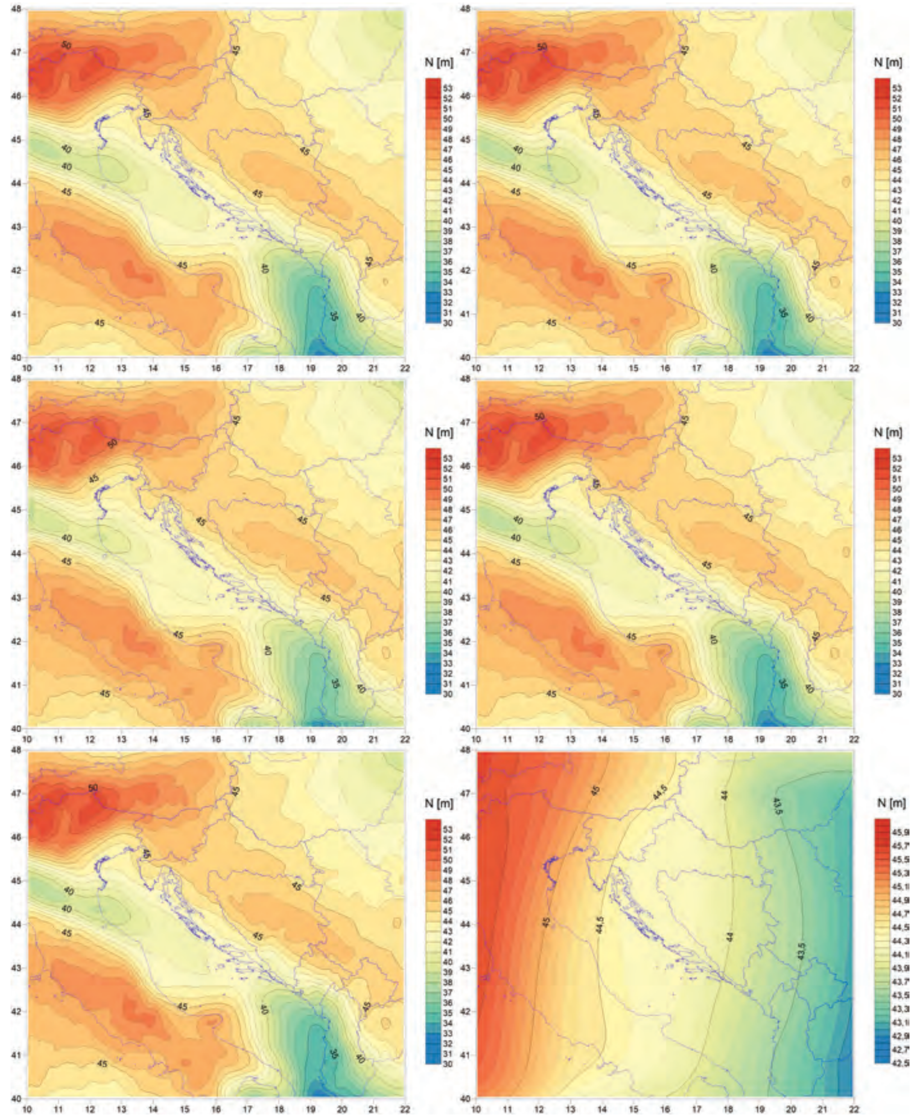

Figure 1: Models of the geoid using GECO and different methods of interpolation, from top left to bottom right: IDW, Kriging, MC, NN, RBF, MA.

Table 3: Modelling parameters

Gridding method	Parameter	Value	Gridding method	Parameter	Value
IDW	Anisotropy ratio	1	Natural Neighbour	Anisotropy ratio	1
	Anisotropy angle	0		Anisotropy angle	0
	SER	7.2	RBF	Anisotropy ratio	1
Kriging	Anisotropy ratio	1		Anisotropy angle	0
	Anisotropy angle	0		SER	7.2
	SER	7.2		SEA	0
Minimum Curvature	Max. Residual	0.02		R^2	0.0006
	Anisotropy Ratio	1	Moving Average	SER	7.2
				SEA	0

Simple visual evaluation of models presented in Figure 1 can lead to some assumptions. Firstly, the surface of the geoid created with the MA method of interpolation seems rather flat and completely out of place in comparison with other models. Secondly, a model created using the MC method shows sudden jumps and roughness of the geoid surface, especially along the edges of the model. The assumption can be made that MA and MC methods will have the lowest level of accuracy in the assessment.

Figures showing geoid models created using EGM2008 and EIGEN-6C4 data aren't shown here as they are very similar to GECO and differences between them can hardly be seen in this kind of resolution. Assumptions made on the GECO models also transfer to the other two.

3 QUALITY ASSESSMENT AND COMPARISON

Newly created or modeled values of geoid undulations $\bar{N}$ and geoid undulations N derived directly from geopotential models in same points allow the calculation of their prediction errors or residuals ε in each of the nodes as

$$\varepsilon_i = N_i - \bar{N}_i. \quad (5)$$

It should be highlighted that none of the undulations N was included in the modelling of $\bar{N}$. This enables the calculation of RMSE as a measure of quality of the created model as (Willmott and Matsuura, 2005)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \varepsilon_i^2}{n}}, \quad (6)$$

where n is a total number of nodes in the grid model.

18 sets of residuals were made in total, one for each of the 18 created grid models, by subtracting the corresponding of the three source models as shown in (5). These sets of residuals also form grid models, as each of the residuals ε has a corresponding planar position of the node it is calculated from. Surfaces of 6 of those models made from GECO, one for each of the tested methods of interpolation, are shown in Figure 2.

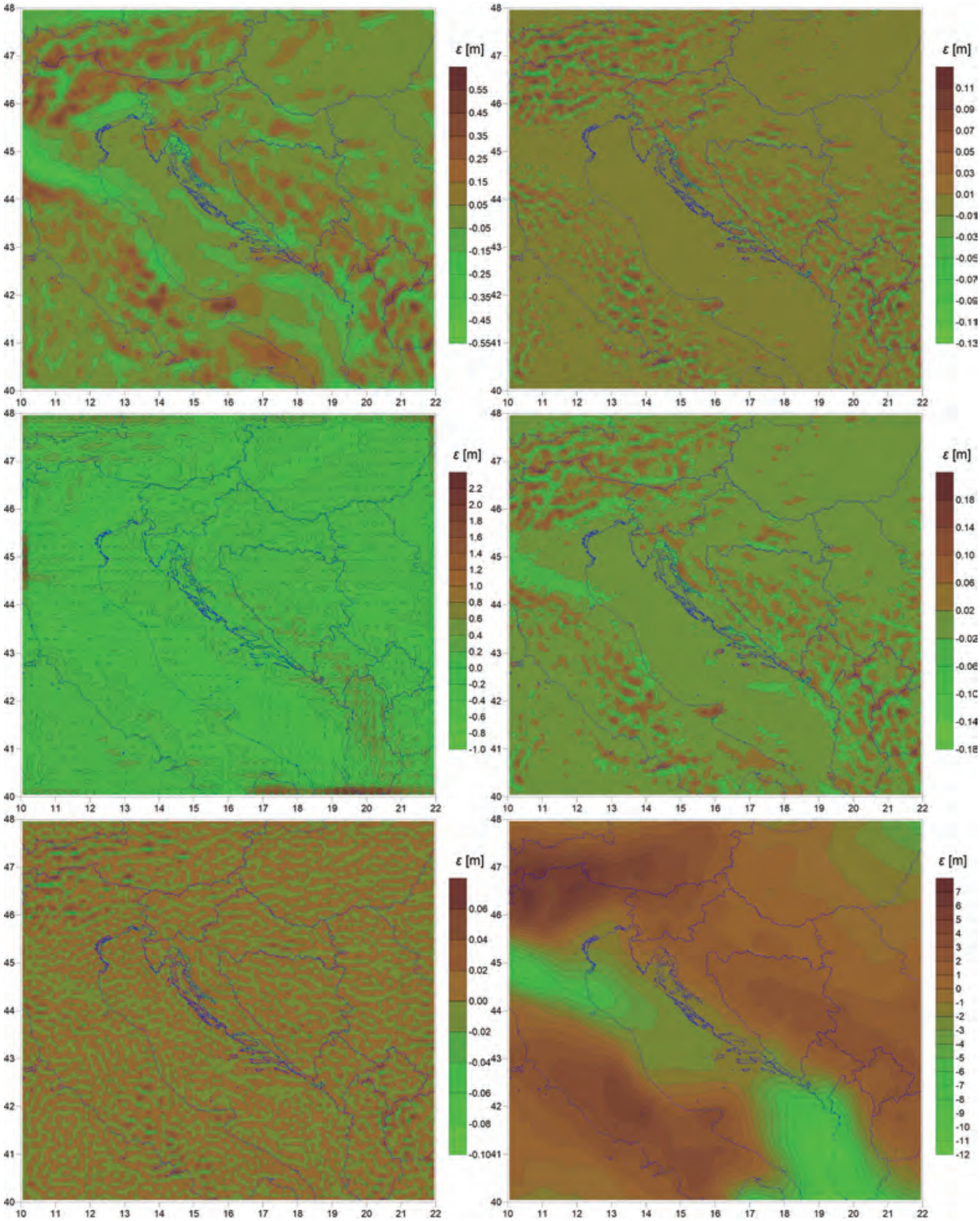

Figure 2: Contour maps showing residuals ε of different methods of interpolation using GECO, from top left to bottom right: IDW, Kriging, MC, NN, RBF, MA.

For the same reason as per Figure 1, contour maps of residuals for EGM2008 and EIGEN-6C4 aren't shown because of their barely noticeable differences from GECO residuals on Figure 2, at least in such a limited resolution. Basic statistical indicators for all 18 sets of residuals are presented in Table 4.

Table 4: Basic statistical indicators of residuals ε depending on the tested method of interpolation and used global geopotential model.

Indicator [m]	Inverse Distance Power	Kriging	Minimum Curvature	Natural Neighbour	Radial Basis Function	Moving Average	Model
ε minimum	-0.548	-0.129	-0.970	-0.173	-0.096	-11.138	EGM2008
ε maximum	0.542	0.104	2.018	0.176	0.061	6.520	
ε span	1.090	0.233	2.987	0.350	0.157	17.658	
ε mean	0.001	0.000	0.017	0.001	0.000	-0.009	
ε median	-0.004	0.000	0.009	-0.001	0.000	0.418	
RMSE	0.102	0.018	0.178	0.027	0.012	2.940	
ε minimum	-0.549	-0.129	-0.951	-0.173	-0.096	-11.199	EIGEN-6C4
ε maximum	0.545	0.104	2.036	0.176	0.061	6.538	
ε span	1.093	0.233	2.988	0.349	0.157	17.737	
ε mean	0.001	0.000	0.017	0.001	0.000	-0.007	
ε median	-0.004	0.000	0.009	-0.001	0.000	0.424	
RMSE	0.102	0.018	0.179	0.027	0.012	2.950	
ε minimum	-0.547	-0.129	-0.974	-0.173	-0.096	-11.192	GECO
ε maximum	0.540	0.104	2.035	0.176	0.061	6.513	
ε span	1.087	0.233	3.009	0.349	0.157	17.705	
ε mean	0.001	0.000	0.017	0.001	0.000	-0.007	
ε median	-0.005	0.000	0.009	-0.001	0.000	0.420	
RMSE	0.102	0.018	0.179	0.027	0.012	2.949	

Figure 2 shows noticeable differences in spatial distribution of residuals for MC and MA in comparison with other methods of interpolation, which confirms previous assumptions. Inspection of information provided in Table 4 also agrees with this. All of the indicators for MA except the mean residual are on the meter level and drastically worse than those of other methods. Indicators for MC are also somewhat worse than the remaining 4 methods, RMSE and span of residuals being the highest and mean residual being furthermost away from zero. Although these indicators for MC aren't drastically lower than for example IDW, in combination with specific spatial distribution noticeable on Figure 2 they lead to the conclusion that method of MC isn't suitable for interpolation of geoid undulations from grid models.

Another thing that can be noted by examination of Table 4 is a high level of similarity of all expressed indicators between different global geopotential models. Differences between corresponding indicators are on average only a few mm high, or to be exact, with omitting the MA and MC methods, highest difference in any indicator between models is only 6 mm. In other words, results for EGM2008, EIGEN-6C4 and GECO are to a large degree the same if observing the significant cm level. This implies a high level of similarity between the geoid models created from those different global geopotential models and enables making of conclusions that are carried over for all of the models.

To make a comparison of indicators between methods of interpolation from Table 4 easier, they are graphically represented in Figure 3 for GECO only, as ones for EGM2008 and EIGEN-6C4 are almost the same.

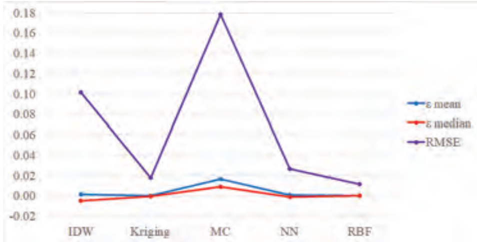

Figure 3: Comparison of RMSE, mean and median ε for different methods of interpolation and GECO model

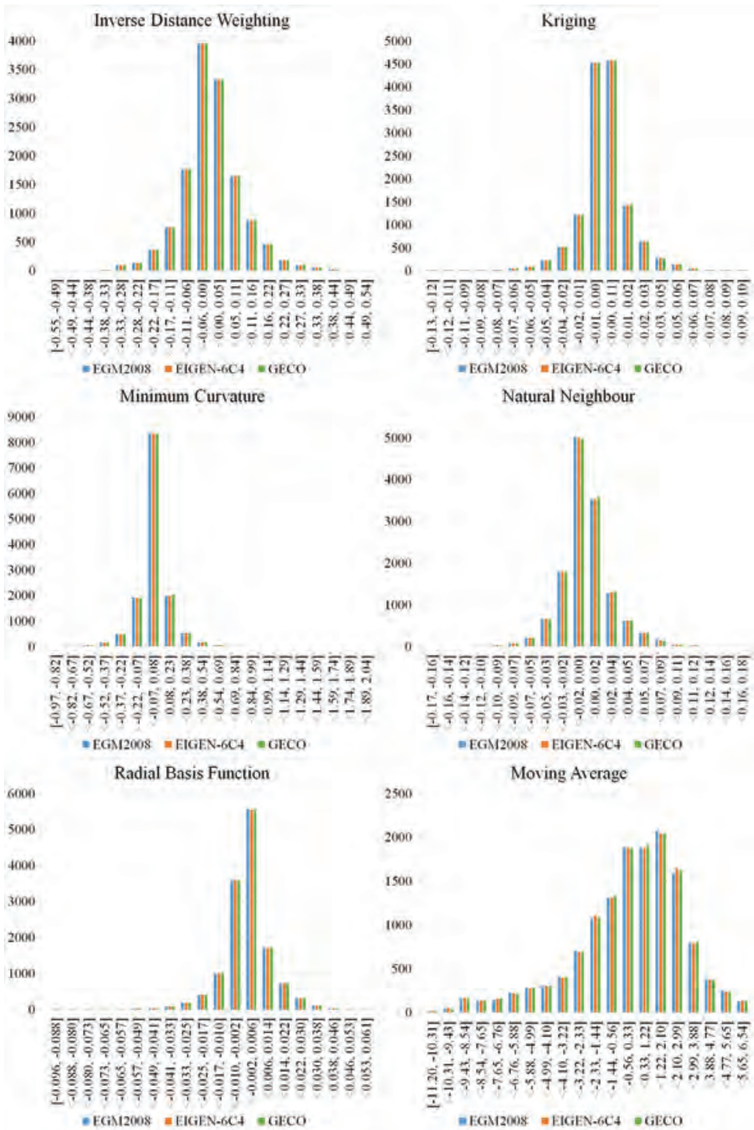

Figure 4: Histograms of residuals ε for all of the tested interpolation methods

Figure 3 confirms high deviation of MC method and also gives insight in noticeable deviation of IDW with RMSE of 10.2 cm and median ε of -0.5 cm. RMSE of remaining three methods are all considerably better with amounts: 1.8 cm for Kriging, 2.7 cm for NN and 1.2 cm for RBF. They also have means and medians of ε equal or almost equal to zero (0.0 or 0.1 cm). Maximum and minimum residuals with the corresponding span are also following this trend (Table 4), with RBF method having those indicators closest to zero and Kriging and NN following closely behind.

Another indicator that should be regarded while comparing interpolation quality is the distribution of different interpolation method's residuals. Histograms of all calculated residuals showing this distribution are presented in Figure 4, for all tested methods and global geopotential models. They are created with 20 equal-sized intervals or bins.

All of the distributions of residuals (Figure 4), except residuals of MA methods, are reasonably bell-shaped, symmetric and without outliers. MC method dispersion is rather narrow and is heavily gravitating to zero value, and the histogram is in addition right-skewed. Dispersions of IDW, Kriging, NN, and RBF are to some extent resembling normal distributions, with RBF being slightly skewed left.

4 SUMMARY

In this paper, the authors determined and compared the accuracy of six spatial interpolation methods in calculating geoid undulations from equiangular grid models of the geoid, derived from three different global geopotential models. For this, 18 grid models of the geoid were created, and modelled values of geoid undulations $\overline{N}$ from these models were subtracted from undulations N of independent (e.g. not used in creation) models in equal points. Residuals calculated in this way, their statistics and their graphical interpretation, served as a base for quality assessment and comparison of six tested methods of interpolation.

The first thing that was noted is the very high level of similarity between different global geopotential models. Statistical and graphical indicators are virtually the same within each of the different methods of interpolation and seem independent of global model in question. This indicates that EGM2008, EIGEN-6C4 and GECO are extremely similar on the observed wide area of Croatia (10°-22° λ and 40°-48° φ on GRS80).

Comparative analysis of interpolation methods led to the conclusion that, for the specific case of interpolation of geoid undulation from grid model, MA method is completely unsuitable. MC and IDW also turned out to have a low level of accuracy and are not suited for this purpose. Based on all of the considered criteria, it can be concluded that best methods are IDW, Kriging, and NN, in that order. For these three methods, differences in all of the considered indicators were in the range of a few mm, and all of them should give satisfactory results on cm level. It should be noted that methods were tested using default interpolation parameters, so the possibility of obtaining different results by tweaking them exist.

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