UPORABA VARIACIJ POSPEŠKA ZARADI VPLIVA LUNE IN SONCA PRI IZRAČUNU TIRNIC SATELITOV GLONASS

IMPLEMENTATION OF THE VARIATION OF THE LUNI-SOLAR ACCELERATION INTO GLONASS ORBIT CALCULUS

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IZVLEČEK

V sistemu diferencialnih enačb, ki opisuje gibanje satelitov GLONASS (rus. Globalnaya Navigazionnaya Sputnikovaya Sistema*), se pospešek, ki ga povzroča vpliv Lune in Sonca pogosto upošteva kot konstanta v obdobju integracije. V pričujoči raziskavi predpostavljamo, da pospešek zaradi vpliva Lune in Sonca ni konstanten, temveč se v obdobju integracije linearno spreminja. Po tej predpostavki izračunamo linearne funkcije v treh oseh pospeška zaradi vpliva Lune in Sonca za interval 30 minut in jih nato implementiramo v diferencialne enačbe. V dokumentu GLONASS-ICD (angl.* Interface Control Document*) je za določitev tirnice priporočena uporaba numerične integracije Runge-Kutta četrtega reda za reševanje sistema diferencialnih enačb. Izračun položaja in hitrosti satelita GLONASS je v tej študiji izveden z metodo Runge-Kutta četrtega reda pri integraciji naprej in nazaj, pri čemer so začetni pogoji navedeni v datoteki s satelita oddanih efemerid.*

ABSTRACT

In the differential equation system describes the motion of GLONASS satellites (rus. Globalnaya Navigazionnaya Sputnikovaya Sistema*, or Global Navigation Satellite System), the acceleration caused by the luni-solar traction is often taken as a constant during the period of the integration. In this work-study, we assume that the acceleration due to the luni-solar traction is not constant but varies linearly during the period of integration following this assumption; the linear functions in the three axes of the luni-solar acceleration are computed for an interval of 30 min and then implemented into the differential equations. The use of the numerical integration of Runge-Kutta fourth-order is recommended in the GLONASS-ICD (Interface Control Document) to solve for the differential equation system in order to get an orbit solution. The computation of the position and velocity of a GLONASS satellite in this study is performed by using the Runge-Kutta fourth-order method in forward and backward integration, with initial conditions provided in the broadcast ephemerides file.*

KLJUČNE BESEDE

pospešek zaradi vpliva Lune in Sonca, oddajanje efemerid, metoda Runge-Kutta, integracija naprej, integracija nazaj

SI | EN

1 INTRODUCTION

GLONASS stands for Globalnaya Navigazionnaya Sputnikovaya Sistema, or Global Navigation Satellite System was developed by the USSR (Union of Soviet Socialist Republics) in 1976 and is now operated by Russia, with the first GLONASS satellite launched on 12 October, 1982, the full constellation was completed and put into operation at the beginning of 1996 (Guoet et al., 2020). Like GPS (Global Positioning System), GLONASS is used to provide positioning, velocity and precise time for marine, aerial, terrestrial, and space users (ICD, 2016).

The space segment of GLONASS consists of 23 operational satellites and three backup satellites, which are evenly distributed over three orbital planes with an inclination of 64.8°. The longitude of the ascending node of each plane differs by 120° from plane to plane. In each orbit plane, there are eight satellites separated by 45° in the argument of latitude, the altitude of a satellite above the terrestrial surface is 19100 km and its orbital period is 11 h 15 min 44 s (Guo et al., 2020). In September 2016, the number of operational satellites in orbit was increased to 27, 24 of which are GLONASS-M and GLONASS-K1 satellites with full operational capability. The GLONASS broadcast ephemerides are given in the PZ-90 (Parametry Zemli 1990 or Parameters of the Earth 1990) reference frame (ICD, 2016).

Using the Runge-Kutta method, there are two assumptions in computing the orbit of GLONASS satellite; the first assumption consists of supposing the acceleration of the GLONASS satellite due to the luni-solar traction is constant during an interval of integration, this assumption is commonly employed for the computation of GLONASS satellites positions and velocities. The second assumption supposes this acceleration varies linearly during 30 min.

The first and second assumption has been presented in (Lin et al., 2009), where the authors compared the results between two different orders of Runge-Kutta integration: the 4th order for the first assumption and order 4 and 5 using the ODE45 solver (Ordinary Differential Equation) directly for the second assumption. In addition, the calculus of GLONASS orbits in (Lin et al., 2009) was made in forward integration only, in an interval of 30 min, and the results were compared with the coordinates of the broadcast ephemeris. Note that the ODE45 is a MATLAB built-in function based on the Runge-Kutta integration method of order 4 and 5, a method that was presented by Dormand and Prince. For more details, see Dormand and Prince (1980), Mathworks (2020).

The main objective of this work is to consider the variation of the luni-solar acceleration into the differential equation system of GLONASS satellite motion (assumption 2) such that we respect the recommendations of ICD document by using the 4^{th}order Runge-Kutta (RK04) method and the 15 min integration interval in forward and backward process. The forward and backward integration in the two assumptions by RK04 are repeated for 48 epochs during 24 hours, and the averaged numerical results are presented and compared. Effectively, a comparison of our results with the precise orbits is highly desired. However, we know that the precise orbits refer to the Center of Mass (CoM) of the satellites, not to the Phase Center (PC) as the broadcast orbits refer to. For this reason, we could not make this comparison for the GLONASS satellites in the present paper.

The present study is organized as follows: In section 2, the differential equation system of motion of a GLONASS satellite is first given. An extract of a GLONASS ephemerides file is described as well as how to compute the variation of luni-solar acceleration. In section 3, a detailed review of how to solve a dif-

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ferential equation system by RK04is presented. Subsequently, in section 4, we give the definition of the forward and the backward integration techniques, which are implemented in our calculus for comparison purposes. Section 4 is also devoted to providing the numerical results and their analysis corresponding to the implementation of both assumptions stated above. Finally, a summary of conclusions is given.

2 GLONASS SATELLITE MOTION

2.1 Equation of motion

The motion of a GLONASS satellite orbiting around the Earth is described by the following system: Biron (2001), ICD (2016):

$$\begin{cases} \dfrac{dV_x}{dt} = -\dfrac{GM}{r^3}X + \dfrac{3}{2}C_{20}GM\dfrac{a^2}{r^5}\left(1 - \dfrac{5Z^2}{r^2}\right)X + w^2X + 2wVy + \gamma_x \\[3mm] \dfrac{dV_y}{dt} = -\dfrac{GM}{r^3}Y + \dfrac{3}{2}C_{20}GM\dfrac{a^2}{r^5}\left(1 - \dfrac{5Z^2}{r^2}\right)Y + w^2Y + 2wVx + \gamma_y \\[3mm] \dfrac{dV_z}{dt} = -\dfrac{GM}{r^3}Z + \dfrac{3}{2}C_{20}GM\dfrac{a^2}{r^5}\left(3 - \dfrac{5Z^2}{r^2}\right)Z + \gamma_z \end{cases} \qquad (1)$$

where G is the gravitational constant, M is the mass of Earth, r is the orbital radius, C_{20} is the second-order harmonic coefficient, a is the equatorial radius of Earth, X, Y, Z are the coordinates of satellite, V_x, V_y, V_z are velocities of satellites with respect to the three axes, and $dV_x/dt, dV_y/dt, dV_z/dt$ are their temporal variation, w is the rotation rate of Earth, $\gamma_x \gamma_y \gamma_z$ the acceleration due to the luni-solar perturbation.

2.2 Broadcast Ephemerides

The GLONASS broadcast orbits are updated each 30 min (Biron, 2001; Cheng, 1998). Starting from the ephemerides at the time of reference t_b, the users can then predict and compute the satellite ephemerides and velocities at time t_c.

Table 1 is an example of GLONASS broadcast ephemerides extracted from the navigation file. Table 1 contains two records of GLONASS navigational message (Broadcast Ephemerides) in RINEX format (*Receiver INdependent Exchange*). It relates to the GLONASS SV 07 from 9th February 2020 at 11 h 45 m and 12 h 15 min time of GLONASS.

Table 1:   Example of GLONASS ephemerides file for GLONASS SV 07

```
R07 2020 02 09 11 45 00 -.329930335283E-04 -.909494701773E-12  .414000000000E+05
    .124674428711E+05 -.237827777863E+01  .558793544769E-08 .000000000000E+00
    .126838168945E+05 -.794471740723E+00  .000000000000E+00 .500000000000E+01
   -.182341059570E+05 -.217863845825E+01  .000000000000E+00 .000000000000E+00
R07 2020 02 09 12 15 00 -.329948961735E-04 -.909494701773E-12  .432000000000E+05
    .768464257813E+04 -.289616584778E+01  .465661287308E-08 .000000000000E+00
    .114883041992E+05 -.504366874695E+00 -.931322574615E-09 .500000000000E+01
   -.213969350586E+05 -.131273841858E+01  .931322574615E-09 .000000000000E+00
```

In Table 1, the first three columns after each header line designate respectively: the coordinates (km), the velocities (km/s) and the luni-solar accelerations (km/s²) (see also Gurtner, 2007; Maciuk, 2016).

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The broadcast ephemerides file of GLONASS SV 07 used in this work study was downloaded from the IGS(International GNSS Service) database.

2.3 Calculation of the variation of the luni-solar acceleration

The effects of the gravitational attraction of the sun and the moon on the orbit of an artificial satellite have been studied in several papers. For more details, see Solórzano and Prado (2013).

In this study, we assume, as previously mentioned, that the luni-solar acceleration follows a linear variation during an interval of 30 min between two epochs of reference of broadcast ephemerides t_{b1} and t_{b2} as follows:

$$\begin{cases} \gamma_x = a_x t + b_x \\ \gamma_y = a_y t + b_y \\ \gamma_z = a_z t + b_z \end{cases} \tag{2}$$

With: $\begin{cases} a_x = (\gamma_x(t_{b2}) - \gamma_x(t_{b1}))/(t_{b2} - t_{b1}) \ and \ b_x = \gamma_x(t_{b1}) - a_x t_{b1} \\ a_y = (\gamma_y(t_{b2}) - \gamma_y(t_{b1}))/(t_{b2} - t_{b1}) \ and \ b_y = \gamma_y(t_{b1}) - a_y t_{b1} \\ a_z = (\gamma_z(t_{b2}) - \gamma_z(t_{b1}))/(t_{b2} - t_{b1}) \ and \ b_z = \gamma_z(t_{b1}) - a_z t_{b1} \end{cases}$ (3)

Such that: t_{b1} and t_{b2} are the epochs of reference, $\gamma_x \gamma_y$ and γ_z are as defined in the broadcast ephemerides file. The following figure represents the accelerations due to the luni-solar traction with respect to the three axes.
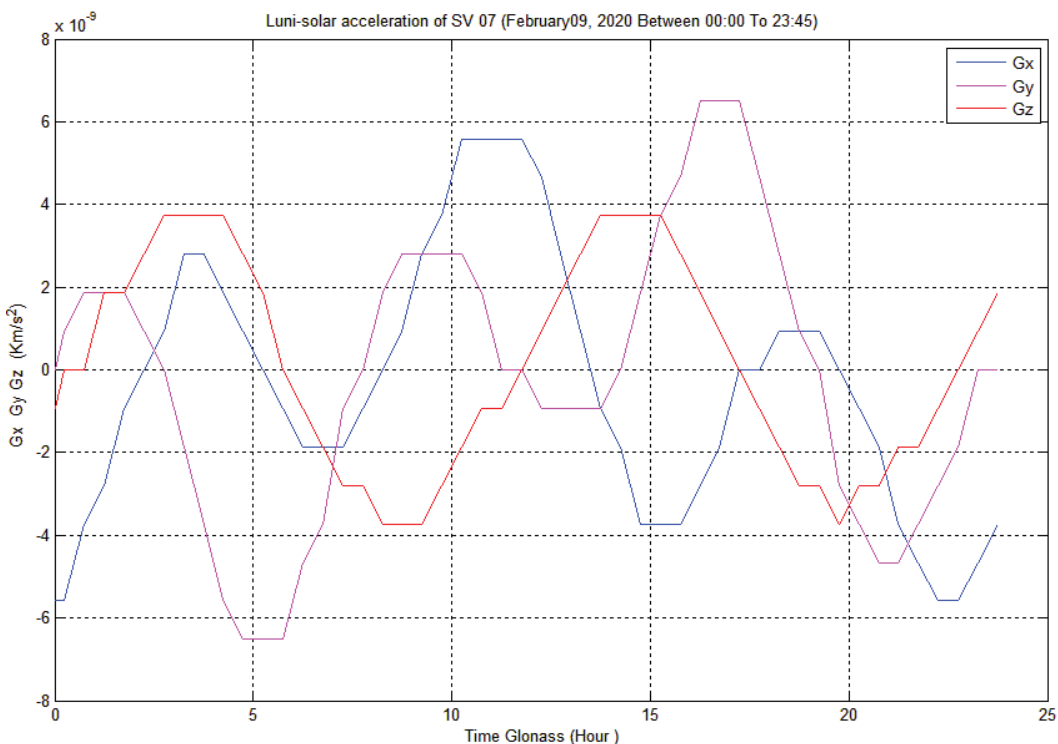

Figure 1: Luni-solar acceleration of GLONASS SV 07 (09/02/2020).

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Due to the luni-solar traction, these accelerations are recorded on February 09, 2020 for GLONASS SV 07 from the IGS database.

3 RUNGE-KUTTA METHOD INTEGRATION (RK)

3.1 General principle of RK integration

The Runge-Kutta integration is, in principle, a numerical resolution of a first-order differential equation. This method allows us to compute an approximate value of y_{i+1} from a previous value y_i (Maciuk, 2016). As Runge-Kutta integration is designed to first-order equations, the second-order differential equation system (1) in hand should be transformed to a first-order differential equation system, this operation gives the following:

$$
\begin{cases}
\begin{cases}
\dot{X} = V_x \\
\dot{Y} = V_y \\
\dot{Z} = V_z
\end{cases} \\
\begin{cases}
\dot{V}_x = -\dfrac{GM}{r^3}X + \dfrac{3}{2}C_{20}GM\dfrac{a^2}{r^5}\left(1-\dfrac{5Z^2}{r^2}\right)X + w^2X + 2wV_y + \gamma_x \\
\dot{V}_y = -\dfrac{GM}{r^3}Y + \dfrac{3}{2}C_{20}GM\dfrac{a^2}{r^5}\left(1-\dfrac{5Z^2}{r^2}\right)Y + w^2Y + 2wV_x + \gamma_y \\
\dot{V}_z = -\dfrac{GM}{r^3}Z + \dfrac{3}{2}C_{20}GM\dfrac{a^2}{r^5}\left(1-\dfrac{5Z^2}{r^2}\right)Z + \gamma_z
\end{cases} \\
\begin{cases}
\dot{\gamma}_x = 0 \ (\gamma_x \text{ is constant}) \\
\dot{\gamma}_y = 0 \ (\gamma_y \text{ is constant}) \\
\dot{\gamma}_z = 0 \ (\gamma_z \text{ is constant})
\end{cases}\text{Assumption 1}
\begin{cases}
\dot{\gamma}_x = a_x \ (\gamma_x \text{ varies linearly}) \\
\dot{\gamma}_y = a_y \ (\gamma_y \text{ varies linearly}) \\
\dot{\gamma}_z = a_z \ (\gamma_z \text{ varies linearly})
\end{cases}\text{Assumption 2}
\end{cases}
\tag{4}
$$

Equation (4) can be written in the general form: $dY = f(t, Y)$ and $Y = [X, Y, Z, Vx, Vy, Vz, \gamma x, \gamma y, \gamma z]^T$.

The values of $(\gamma_x, \gamma_y, \gamma_z)$ in the above equation system are usually taken as constant (Assumption 1) when attempting to solve for the vector of parameters Y by Runge-Kutta method.

3.2 Fourth order Runge-Kutta method (RK04)

The computation of the position (y_{i+1}), the (v_{i+1}) velocity and (γ_{i+1}) the acceleration luni-solar at iteration $(i+1)$ is carried out from the knowledge of (y_i), (v_i), and (γ_i) at the previous iteration (i), according to these formulas (Maciuk, 2016; Cheng, 1998):

$$
\begin{cases}
y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)/6 \\
v_{i+1} = v_i + (m_1 + 2m_2 + 2m_3 + m_4)/6 \\
\gamma_{i+1} = \gamma_i + (g_1 + 2g_2 + 2g_3 + g_4)/6
\end{cases}
\tag{5}
$$

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For our study, the coefficients k_i, m_i and g_i are given by:

$$k_1 = \begin{cases} k_1 = hf(t_n, y_n, v_n, \gamma_n) \\ k_2 = hf(t_n + h/2, y_n + k_1/2, v_n + m_1/2, \gamma_n + g_1/2) \\ k_3 = hf(t_n + h/2, y_n + k_1/2, v_n + m_1/2, \gamma_n + g_1/2) \\ k_4 = hf(t_n, y_n, v_n, \gamma_n) \end{cases} \qquad (6)$$

$$m_1 = \begin{cases} m_1 = hf(t_n, y_n, v_n, \gamma_n) \\ m_2 = hf(t_n + h/2, y_n + k_1/2, v_n + m_1/2, \gamma_n + g_1/2) \\ m_3 = hf(t_n + h/2, y_n + k_1/2, v_n + m_1/2, \gamma_n + g_1/2) \\ m_4 = hf(t_n, y_n, v_n, \gamma_n) \end{cases} \qquad (7)$$

$$g_1 = \begin{cases} g_1 = hf(t_n, y_n, v_n, \gamma_n) \\ g_2 = hf(t_n + h/2, y_n + k_1/2, v_n + m_1/2, \gamma_n + g_1/2) \\ g_3 = hf(t_n + h/2, y_n + k_1/2, v_n + m_1/2, \gamma_n + g_1/2) \\ g_4 = hf(t_n, y_n, v_n, \gamma_n) \end{cases} \qquad (8)$$

The position, velocity and acceleration luni-solar are given by:

$$\begin{cases} y_i = \begin{cases} x_{i+1} = x_i + h(k_{11} + 2k_{21} + 2k_{31} + k_{41})/6 \\ y_{i+1} = y_i + h(k_{12} + 2k_{22} + 2k_{32} + k_{42})/6 \\ z_{i+1} = z_i + h(k_{13} + 2k_{23} + 2k_{33} + k_{43})/6 \end{cases} \\ v_i = \begin{cases} v_{x,i+1} = v_{x,i} + h(m_{11} + 2m_{21} + 2m_{31} + m_{41})/6 \\ v_{y,i+1} = v_{y,i} + h(m_{12} + 2m_{22} + 2m_{32} + m_{42})/6 \\ v_{z,i+1} = v_{z,i} + h(m_{13} + 2m_{23} + 2m_{33} + m_{43})/6 \end{cases} \\ \gamma_i = \begin{cases} \gamma_{x,i+1} = \gamma_{x,i} + h(g_{11} + 2g_{21} + 2g_{31} + g_{41})/6 \\ \gamma_{y,i+1} = \gamma_{y,i} + h(g_{12} + 2g_{22} + 2g_{32} + g_{42})/6 \\ \gamma_{z,i+1} = \gamma_{z,i} + h(g_{13} + 2g_{23} + 2g_{33} + g_{43})/6 \end{cases} \end{cases} \qquad (9)$$

In all equations, h is the integration step; the formulas for calculating k_i, m_i and g_i are provided in the appendix.

4 PROCESS OF GLONASS ORBIT CALCULATION

4.1 Results relative to one epoch (11:45 to 12:15)

Based on the known initial values of position (X_0, Y_0, Z_0), velocity (Vx_0, Vy_0, Vz_0), and luni-solar accelera-tion (γx_0, γy_0, γz_0), given in the broadcast ephemerides file, it is possible to determine satellite's position, velocity and the luni-solar acceleration for any moment within the range ±15 min.

Using the information given in (Table 1), the position, velocity and luni-solar acceleration of GLO-NASSSV 07 at time 12:00 can be calculated as follows:

– **In Forward**: The orbit of SV 07 between 11:45 to 12:00 can be interpolated from the initial values given in broadcast ephemerides file at time t_{b1} = 11:45

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– **In Backward**: The orbit of SV 07 between 12:00 to 12:15 can be integrated from the initial values given in broadcast ephemerides at time t_{b2} = 12:15

Figure 2 illustrates the process of GLONASS orbits computation in forward and backward integration for one epoch.
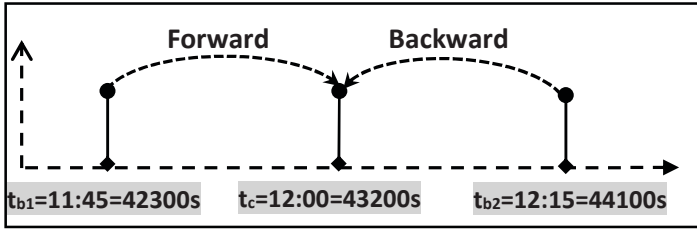

Figure2:   Forward and backward integration process.

To compare the numerical results, we made the two following assumptions:

– **The 1st Assumption**: The terms $\gamma_x \gamma_y$ and γ_z constant during 15min.
– **The 2nd Assumption**: The terms $\gamma_x \gamma_y$ and γ_z vary linearly during 15min (c.f sec2.3).

Using RK04 method with an integration step $h=0.01s$ in the computation, the Table2 and Figure3 shows the differences at time t_c=12:00 between the 1st and 2nd assumption.

Table 2:   Differences between the 1st and 2nd assumption.

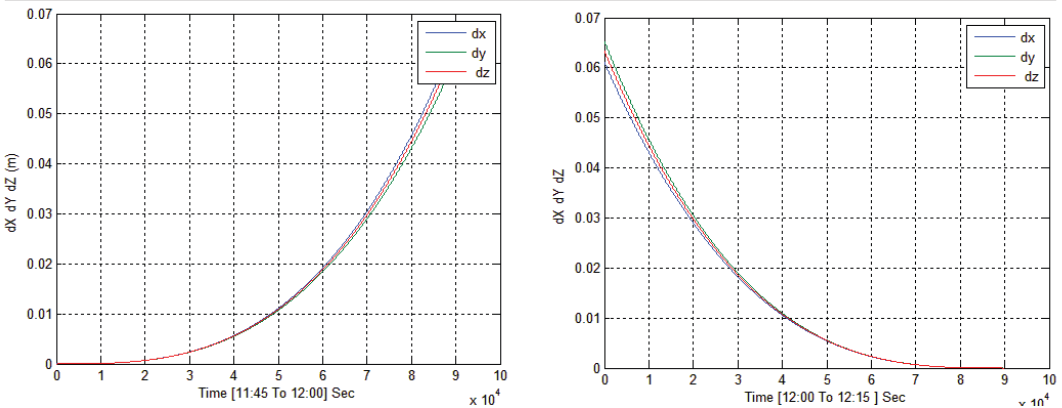|	Assumption 1 $d_1 = \lvert \vec{Y}_{Fwd} - \vec{Y}_{Bwd} \rvert$	Assumption 2 $d_2 = \lvert \vec{Y}_{Fwd} - \vec{Y}_{Bwd} \rvert$	$d = \lvert d_1 - d_2 \rvert$
dX(m)	2,333	2.459	0.126
dY(m)	1.755	1.628	0.127
dZ(m)	3.219	3.346	0.127
dVx(m)	0.00128173	0.00126323	0.0000185
dVy(m)	0.00148306	0.00150145	0.0000183
dVz(m)	0.00077236	0.00077238	0.0000000

Figure 3:   Difference between forward (figure left) and backward(figure right) integration.

It is worth notice that the results of the second assumption have been calculated by equations (2) and (3), the linear three functions of variation of luni-solar acceleration of SV 07 between 11:45 to 12:15 are:

$$\begin{cases} \gamma_x = +5.174\,10^{-13}\,t + 2.747\,10^{-08} \\ \gamma_y = -5.174\,10^{-13}\,t + 2.188\,10^{-08} \\ \gamma_z = +5.174\,10^{-13}\,t + 2.188\,10^{-08} \end{cases} \tag{10}$$

Figure 4 show the orbit of SV 07 with the 1st assumption and the 2nd assumption.
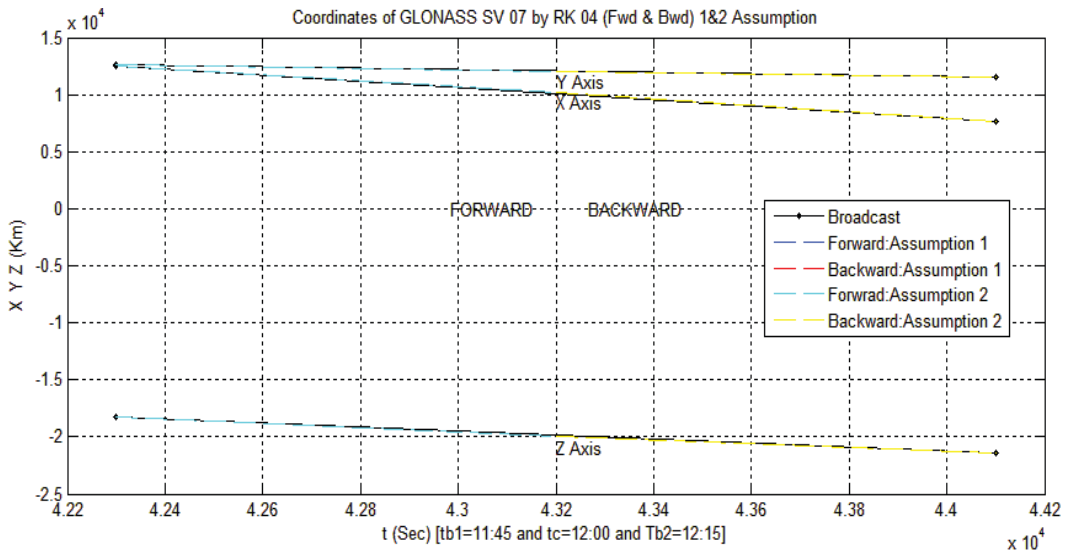
Figure 4: *Coordinates of GLONASS SV 07 (11:45 to 12:15).*

Figure 5 represents more detail from Figure 4 near t_c=12:00.
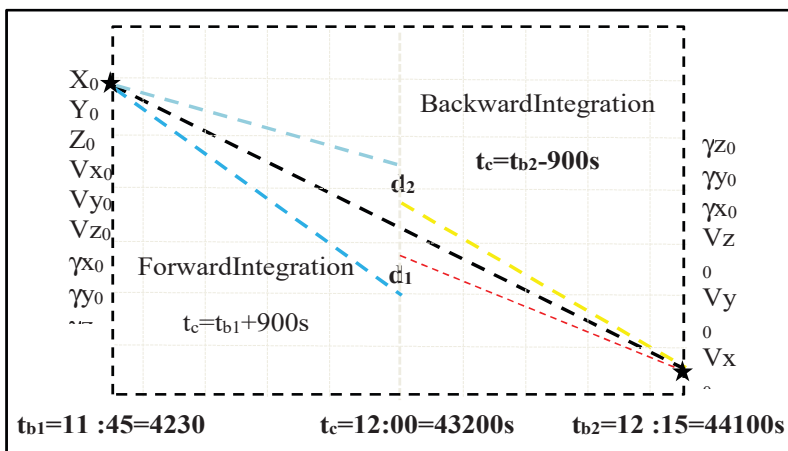

Figure 5:   Detail of epoch t_c=12:00.

Figure 5 clearly shows the shifts between forward, backward and broadcast solutions.

4.2 Results relative to 24 hours (23:45 to 23:45)

Knowing the initial values of position, velocity and luni-solar acceleration at 48 different epochs (24 hours, 23:45 to 23:45) from the broadcast ephemerides of GLONASS SV 07 unregistered on **February 09, 2020** we can compute the positions, velocities and luni-solar accelerations at *(H: 00)* and also *(H:30),* where *H* is Hour. This operation is illustrated in Figure 6.
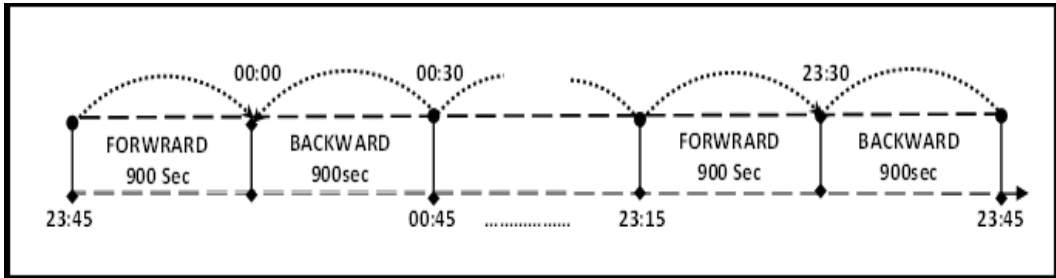

Figure 6: Scheme of the forward and backward integration for 48 epochs =24hours.

The computation of the parameters (*Y, V* and *γ*) by the 4th order Runge-Kutta method, the statistics of differences in position and velocity between forward and backward integration for the 1st and the 2nd assumption for the twenty-four hours period are shown in the following table and figure.

Table 3:   The statistics of differences (Forward-Backward) over 24 hours.

	1st Assumption			2nd assumption		
	Min	**Max**	**Mean**	**Min**	**Max**	**Mean**
dX (m)	0.057	4.913	2.136	0.0160	4.917	2.143
dY(m)	0.327	4.328	2.282	0.210	4.429	2.293
dZ(m)	0.136	5.567	3.148	0.084	5.503	3.198
dVx(m/s)	0.0000012	0.0037806	0.0012028	0.0000383	0.0040186	0.0011962
dVy (m/s)	0.0000055	0.0037806	0.0006797	0.0000216	0.0021952	0.0007012
dVz (m/s)	0.0000620	0.0021584	0.0012918	0.0000567	0.0032829	0.0012567

We can deduce from Table3 some remarks:

First, when comparing forward and backward procedures, the maximal value can reach 5.567 m in the 1st assumption and 5.503 m in the 2nd assumption on the z-axis.

Effectively, this small difference with a slight priority of the second assumption is not significant compared to the GLONASS orbit dimensions (Semi-major axis of GLONASS orbit = 25440 km). In other parts, it does not bring any improvement in the position calculus of a static receiver. However, the difference is more important on the velocity vector since (Figure 8), in this workstudy; the linear functions of the variation of the luni-solar acceleration are all accounted for in the computation of the velocity (see equation (2) and Appendix).

The differences between the forward and backward integration for 1st and 2nd assumptions are due to several factors: the ephemeris errors affecting the initial conditions, the accuracy of Runge-Kutta method and the step size of interpolation by Runge-Kutta method.

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In Figure 7 and Figure 8, the differences between the 1ˢᵗ assumption and 2ⁿᵈ assumption for coordinates and velocities (i.e. *dX, dY, dZ and dV$_X$, dV$_Y$, dV$_Z$*) are represented along 24 hours of integration.
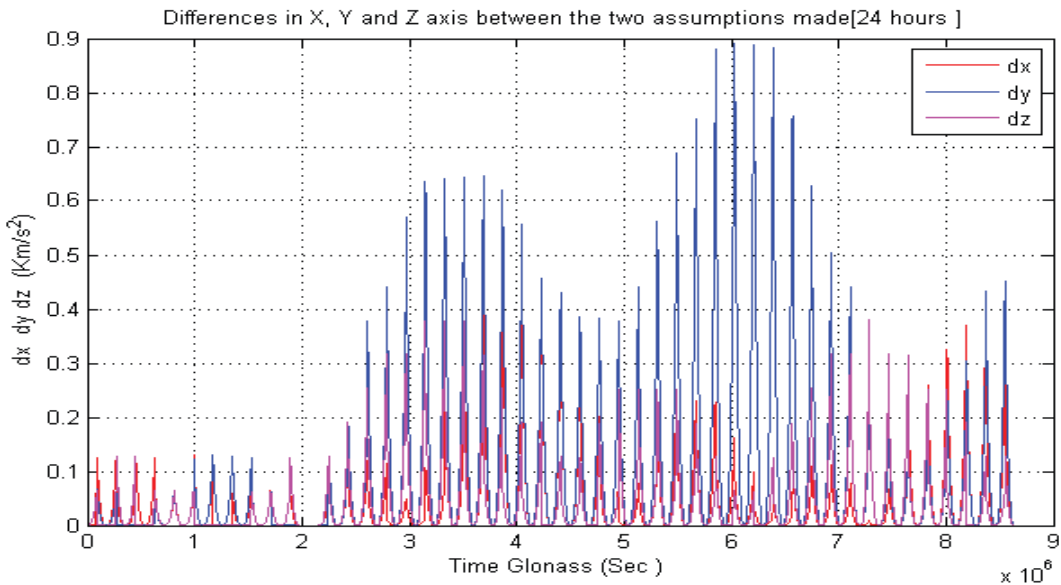

Figure 7:   Differences in X, Y and Z axis between the two assumptions made.
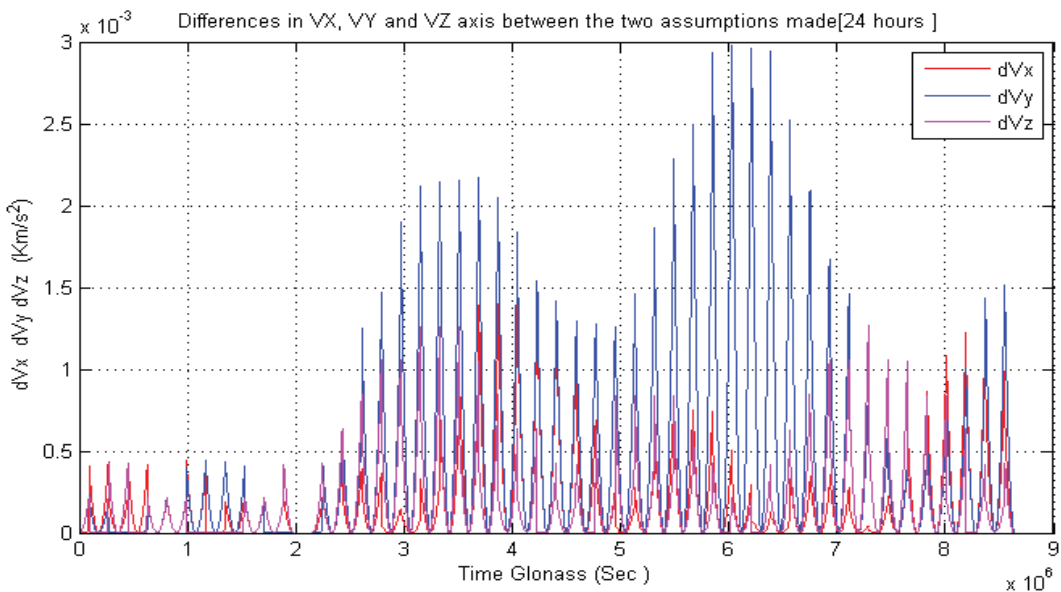

Figure 8:   Differences in V$_x$, V$_y$ and V$_z$ axis between the two assumptions made.

The maximum difference between the two assumptions in coordinates of GLONASS SV 07 is about 1 m and about 0.003km/s in velocity of GLONASS SV 07.

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5 CONCLUSION

The motion of a GLONASS satellite is described by a differential equation system of second order. The resolution of such a mathematical problem is recommended to be done by the 4th order Runge-Kutta method according to the Interface Control Document. In our study, the RK04 method is adopted, the initial conditions and accelerations of GLONASS satellites due to luni-solar traction needed for the resolution of the problem are taken from the GLONASS broadcast navigation file provided by the IGS .

When trying to resolve the differential equation of motion, the luni-solar acceleration affecting the motion of a satellite is often assumed constant during 30 min; this makes our first assumption, whereas our second assumption consists of considering this acceleration varies linearly during 30 min. Besides this, there are two possibilities to compute the orbit solution at a given epoch; the forward integration and the backward integration.

For each assumption, we tested these two possibilities, and we compared their respective results. We conclude from the numerical results that the difference between the main solution of the 1st and 2nd assumption is 1 m in the position of satellite (Y-axis) and about 0.003 km/s in velocity of satellite (Y-axis) over 24 hours. However, for each assumption, the forward solution differs from the backward solution by more than 3 m as maximal value. This difference is due to the ephemeris precision, the order of the Runge-Kutta method and the step size of interpolation.

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APPENDIX : COEFFICIENTS OF RK04 METHOD FOR THE 2^{ED} ASSUMPTION

$$k_{11} = hV_x \quad (A01) \qquad k_{12} = hV_y \quad (A02) \qquad k_{13} = hV_z \quad (A03)$$

$$m_{11} = h \left\{ -\frac{GM}{r^3}(X) + \frac{3}{2}C_0 GM \frac{a^2}{r^5} \left(1 - \frac{5(Z)^2}{r^2} \right)(X) + w^2(X) + 2w(V_y) + (\gamma_x) \right\} \quad (A04)$$

$$m_{12} = h \left\{ -\frac{GM}{r^3}(Y) + \frac{3}{2}C_0 GM \frac{a^2}{r^5} \left(1 - \frac{5(Z)^2}{r^2} \right)(Y) + w^2(Y) + 2w(V_x) + (\gamma_y) \right\} \quad (A05)$$

$$m_{13} = h \left\{ -\frac{GM}{r^3}(Z) + \frac{3}{2}C_0 GM \frac{a^2}{r^5} \left(1 - \frac{5(Z)^2}{r^2} \right)(Z) + (\gamma_z) \right\} \quad (A06)$$

$$g_{11} = ha_x \quad (A07) \qquad g_{12} = ha_y \quad (A08) \qquad g_{13} = ha_z \quad (A09)$$

$$k_{21} = h \left\{ V_x + \frac{m_{11}}{2} \right\} \quad A(10) \qquad k_{22} = h \left\{ V_y + \frac{m_{12}}{2} \right\} \quad (A11) \qquad k_{23} = h \left\{ V_z + \frac{m_{13}}{2} \right\} \quad (A12)$$

$$m_{21} = h \left\{ -\frac{GM}{r^3}\left(X + \frac{k_{11}}{2}\right) + \frac{3}{2}C_{20} GM \frac{a^2}{r^5} \left(1 - \frac{5\left(Z + \frac{k_{13}}{2}\right)^2}{r^2} \right)\left(X + \frac{k_{11}}{2}\right) + w^2\left(X + \frac{k_{11}}{2}\right) + 2w\left(V_y + \frac{m_{12}}{2}\right) + \left(\gamma_x + \frac{g_{11}}{2}\right) \right\} \quad (A13)$$

$$m_{22} = h \left\{ -\frac{GM}{r^3}\left(Y + \frac{k_{12}}{2}\right) + \frac{3}{2}C_{20} GM \frac{a^2}{r^5} \left(1 - \frac{5\left(Z + \frac{k_{13}}{2}\right)^2}{r^2} \right)\left(Y + \frac{k_{12}}{2}\right) + w^2\left(Y + \frac{k_{12}}{2}\right) + 2w\left(V_x + \frac{m_{11}}{2}\right) + \left(\gamma_y + \frac{g_{12}}{2}\right) \right\} \quad (A14)$$

$$m_{23} = h \left\{ -\frac{GM}{r^3}\left(Z + \frac{k_{13}}{2}\right) + \frac{3}{2}C_{20} GM \frac{a^2}{r^5} \left(3 - \frac{5\left(Z + \frac{k_{13}}{2}\right)^2}{r^2} \right)\left(Z + \frac{k_{11}}{2}\right) + \left(\gamma_z + \frac{g_{13}}{2}\right) \right\} \quad (A15)$$

$$g_{21} = h \left\{ a_x + \frac{g_{11}}{2} \right\} \quad (A16) \qquad g_{22} = h \left\{ a_y + \frac{g_{12}}{2} \right\} \quad A(17) \qquad g_{23} = h \left\{ a_z + \frac{g_{13}}{2} \right\} \quad (A18)$$

$$k_{31} = h \left\{ V_x + \frac{m_{21}}{2} \right\} \quad (A19) \qquad k_{32} = h \left\{ V_y + \frac{m_{22}}{2} \right\} \quad A(20) \qquad k_{33} = h \left\{ V_z + \frac{m_{23}}{2} \right\} \quad (A21)$$

$$m_{31} = h \left\{ -\frac{GM}{r^3}\left(X + \frac{k_{21}}{2}\right) + \frac{3}{2}C_{20} GM \frac{a^2}{r^5} \left(1 - \frac{5\left(Z + \frac{k_{23}}{2}\right)^2}{r^2} \right)\left(X + \frac{k_{21}}{2}\right) + w^2\left(X + \frac{k_{21}}{2}\right) + 2w\left(V_y + \frac{m_{22}}{2}\right) + \left(\gamma_x + \frac{g_{21}}{2}\right) \right\} \quad (A22)$$

$$m_{32} = h \left\{ -\frac{GM}{r^3}\left(X + \frac{k_{22}}{2}\right) + \frac{3}{2}C_{20} GM \frac{a^2}{r^5} \left(1 - \frac{5\left(Z + \frac{k_{23}}{2}\right)^2}{r^2} \right)\left(X + \frac{k_{22}}{2}\right) + w^2\left(X + \frac{k_{22}}{2}\right) + 2w\left(V_x + \frac{m_{21}}{2}\right) + \left(\gamma_y + \frac{g_{22}}{2}\right) \right\} \quad (A23)$$

$$m_{33} = h\left\{-\frac{GM}{r^3}\left(Z+\frac{k_{23}}{2}\right)+\frac{3}{2}C_{20}GM\frac{a^2}{r^5}\left(1-\frac{5\left(Z+\frac{k_{23}}{2}\right)^2}{r^2}\right)(Z)+\left(\gamma_z+\frac{g_{23}}{2}\right)\right\} \qquad (A24)$$

$$g_{13} = h\left\{a_x+\frac{g_{21}}{2}\right\} \qquad (A25) \qquad g_{23} = h\left\{a_y+\frac{g_{22}}{2}\right\} \quad (A26) \qquad g_{33} = h\left\{a_z+\frac{g_{23}}{2}\right\} \qquad (A27)$$

$$k_{41} = h\{V_x+m_{31}\} \qquad (A28) \qquad k_{42} = h\{V_y+m_{32}\} \qquad (A29) \qquad k_{43} = h\{V_z+m_{33}\} \qquad (A30)$$

$$m_{41} = h\left\{-\frac{GM}{r^3}(X+k_{31})+\frac{3}{2}C_{20}GM\frac{a^2}{r^5}\left(1-\frac{5(Z+k_{33})^2}{r^2}\right)(X+k_{31})+w^2(X+k_{31})+2w(V_y+m_{32})+(\gamma_x+g_{31})\right\} \qquad (A31)$$

$$m_{42} = h\left\{-\frac{GM}{r^3}(Y+k_{32})+\frac{3}{2}C_{20}GM\frac{a^2}{r^5}\left(1-\frac{5(Z+k_{33})^2}{r^2}\right)(Y+k_{32})+w^2(Y+k_{32})+2w(V_x+m_{31})+(\gamma_y+g_{32})\right\} \qquad (A32)$$

$$m_{43} = h\left\{-\frac{GM}{r^3}(X+k_{33})+\frac{3}{2}C_{20}GM\frac{a^2}{r^5}\left(3-\frac{5(Z+k_{33})^2}{r^2}\right)(Z+k_{33})+(\gamma_z+g_{33})\right\} \qquad (A33)$$

$$g_{41} = h\{a_x+g_{31}\} \qquad (A34) \qquad g_{42} = h\{a_y+g_{32}\} \qquad (A35) \qquad g_{43} = h\{a_z+g_{33}\} \qquad (A36)$$

For the 1ˢᵗ assumption we have; $g_{i,j} = 0$.

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