ALTERNATIVNA METODA TESTIRANJA PREMIKOV V GEODETSKI MREŽI

AN ALTERNATIVE APPROACH TO TESTING DISPLACEMENTS IN A GEODETIC NETWORK

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IZVLEČEK

V geodeziji s postopki statističnega testiranja hipotez z ocenjevanjem in testiranjem značilnih parametrov ugotavljamo izpolnjevanje nekaterih kriterijev in zahtev pri merskih in računskih postopkih. V praksi so se za presojo značilnih premikov uveljavili nekateri kriteriji, ki testno statistiko primerjajo s konstantama $T_{crit} = 3$ *ali* $T_{crit} = 5$. *V članku predlagamo alternativni postopek za določitev empirične porazdelitvene funkcije s simulacijami v 2D- in 3D-geodetski mreži, saj se testna statistika* $T = d/\sigma_d$ *ne porazdeljuje po nobeni od znanih porazdelitvenih funkcij. Kritična vrednost je spremenljivka, ki pri različni dimenziji mreže ob enakem tveganju ni enaka. Na več testnih primerih pokažemo, da je kritično vrednost* T_{crit} *mogoče izračunati natančneje, kar zagotavlja zanesljivo ugotavljanje značilnih premikov glede na izbrano stopnjo značilnosti testa. V vseh testnih primerih* T_{crit} *doseže vrednost 3 pri tveganju, ki je manjše od 1 % za poljubno dimenzijo mreže. Če ocenimo, da je sprejemljivo tveganje 5 %, je kritična vrednost bistveno manjša od 3 ali 5. Značilne premike je torej smiselno obravnavati glede na sprejemljivo tveganje in ne glede na približno ocenjeno kritično vrednost.*

ABSTRACT

In geodesy, statistical testing aids in determining the extent to which the criteria and requirements needed in the measurement and calculation proceedings have been fulfilled. A "rule of thumb" method that compares test statistics to constants $T_{crit} = 3$ *or* $T_{crit} = 5$ *has been established. The test statistic T is the ratio between the displacement and its precision. Since it is not distributed through any of the known distribution functions, (statistical) simulations are used to assess the empirical distribution in 2D and 3D geodetic networks. The proposed alternative procedure leads to a more precise detection of significant displacements at a given test significance level* α. *Regardless of the network's dimensionality,* T_{crit} *obtains the value of 3 at a risk level below 1%. When 5% is considered to be an acceptable risk level, the critical value can be lower than 3 or 5. Thus, significant displacements should be considered with regard to the acceptable risk level and not according to the usual "rule of thumb".*

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statistično testiranje hipotez, stopnja značilnosti, simulirana porazdelitvena funkcija, kritična vrednost, dejansko tveganje, premiki točk

KEY WORDS

statistical hypothesis testing, significance level, simulated distribution function, critical value, actual risk, point displacement

1 INTRODUCTION

In their research, professionals working in the field of geosciences (civil engineers, geologists, miners and others) define the expected displacements. Surveyors can determine the actual displacements based on field measurements such as Marjetič et al. (2012), Kregar et al. (2015), and asses their accuracy using the statistical methodology such as Kregar et al. (2012), Urbančič et al. (2016). This task can also be reversed. We can define the threshold that determines whether the displacement is significant or not with respect to the displacement determination accuracy, which is obtained through measurements and calculations. Some of the approaches can be found in literature. In the literature overview the same symbols as used by the original authors were used.

The classical approach for determining whether the displacement is statistically significant or not is used and described in numerous articles (Caspary, 1987; Dognan et al., 2013; Heck et al., 1977; Koch, 1999; Kuang, 1996; Niemer, 1985, Pelzer, 1971; Savšek-Safić et al., 2003; Sütti and Török, 1996). The critical value at a selected risk level α is compared to the quotient of the quadratic form calculated as the product of the displacement vector and the corresponding cofactor matrix $T = \mathbf{d}^T \Sigma_{\mathbf{dd}}^{-1} \mathbf{d}$, which is distributed by χ_α^2 distribution (Koch, 1999).

Statistical significance of displacement can be tested with conventional deformation analysis (CDA methods) or robust methods (IWST methods). The choice of the method usually depends on the kind of the deformation that is investigated and the type of a control network that is designed for the certain object of study. CDA methods are based on the least squares estimation (LSE) such as Hannover and Karlsruhe method where global congruency test of two epoch adjustment results is used (Pelzer, 1971; Niemeier, 1981). IWST method (Iterative Weighted Similarity Transformation) is a good estimator of the single-point displacement vector in the process of robust S-transformation (Chen et al. 1990). An alternative method M-estimation has been proposed to determine the displacement vector directly from differences in "raw" unadjusted observations (Nowel, 2015; Nowel, 2016). An alternative method is even more efficacious when law values of displacements that slightly exceed measurement errors are expected. A very good empirical measure of the efficacy of outlier tests and robust estimation methods are proposed by Hekimoglu and Koch (2000). The measure is called the mean success rate (MSR) and is computed based on many thousands of different simulated observation sets by the Monte Carlo method as the ratio of the number of sets for which outliers were correctly detected to the number of all sets. The proposed measure of efficiency MSR can be good alternative to statistical measures such as global and local measures of internal reliability in traditional deformation analysis methods.

Rueger (1999) treats the required accuracy of displacement determination similarly to Welsch et al. (2000) and Pelzer et al (1987). If d_y is the estimated value of minimal displacement, then the required accuracy s_y can be assessed with a „rule of thumb" design equation $s_y \leq d/5$. Other authors claim that displacements are significant when the ratio between the displacement and its accuracy is greater than 3 (Klein and Heunecke, 2006).

The U.S. Army Corps of Engineers (USACE Army, 2002) recommends that the standard deviation of measurements for displacement detection should be 9 times lower than the greatest expected displacement. If one uses 95 % confidence level to describe the measurement accuracy, then it should be 4 times lower than the greatest expected displacement.

Hekimoglu et al. (2010) calculate the radius of the corresponding displacement circle r from the elements of the respective sub-matrix of the cofactor matrix $\mathbf{Q_{dd}}$ and α-fractile of the χ^2_2-distribution for 2 degrees of freedom $\chi^2_{2,\alpha}$. They observed what percentage of points has moved in the interval between r and $2r$ or between r and $3r$ for a chosen value of error probability α.

Ramos et al. (2012) developed software for 3D movements of olive trees due to erosion of the terrain. 3D movements were analyzed in two parts: as planimetric and altimetric movements. They decided that the planimetric displacements were significant if the 99 % error ellipses of individual points from each campaign do not intersect. To determine the altimetric movements, they have taken a similar decision: displacement is significant, if the altimetric displacement vector for each point from each campaign is higher than the error interval as calculated by $\sqrt{Error\_C1^2_{99\,\%} + Error\_C2^2_{99\,\%}}$.

So far the literature overview dealt predominantly with displacements in a 2D plane. Berber (2006) and Berber et al. (2009) calculates the threshold value in a 3D space which is an approximation of the 3D confidence ellipsoid using the following equation $\delta_i = \sqrt{\sigma^2_{a_{95_i}} + \sigma^2_{b_{95_i}} + \sigma^2_{h_{95_i}}}$, in which the semi-axes of the 95 % confidence ellipsoid and the vertical interval are obtained as $\sigma_{a,b,h95_i} = 2.795\sigma_{a,b,h_i}$. The standard 3D ellipsoid at $\chi^2_{3;1-\alpha} = 1$ has a confidence level (1-α) of about 20 %. In order to achieve a 95 % confidence level, a multiplication factor 2.795 must be used (Staudinger, 1999).

Displacements and deformations appear on artificial objects such as dams, embankments, bridges, their surroundings (e.g. in reservoir valleys, on the banks of accumulation lakes) as well as in organic areas such as landslides, tectonic faults and marshes. The reasons for displacements can be attributed to external forces (temperature changes, wind, tectonic or seismic effects), mechanical properties of the construction materials and elements, inadequate knowledge of the ground's geomorphologic composition, mechanic properties and hydrological conditions at the time of projecting the object. Establishing the displacements and deformations of artificial and natural objects is one of the most complex tasks in surveying.

Indirectly obtained quantities such as the shift and the corresponding standard deviation (statistically tested with the test statistic $T = d/\sigma_d$) are usually used when assessing the statistical significance of displacements. Since the test statistic T is compared to the critical value T_{crit} – which is determined on the basis of the appropriate distribution function and the chosen significance level – it is important to determine the distribution function correctly. The statistical test determines the subset of a sample space known as the critical value approach or the null hypothesis rejection interval. In practice the criterion of $T > 3$ or $T > 5$, also known as the „rule of thumb" is often used in the assessment of significant displacements. However, as simulation procedures allow us to determine the critical value T_{crit} much more accurately this criterion is not precise enough.

In this article we will deal with the specialities used in the calculation of displacements with corresponding standard deviations in 1D, 2D and 3D geodetic networks. The procedures used to determine the distribution function for the test statistic $T = d/\sigma_d$ are proposed according to the dimensionality of the network. For a 2D and 3D geodetic network the test statistic in not distributed through an analytical distribution function, which is why the appropriate distribution function is determined empirically, through simulations. Since the simulations allow us to determine the critical value T_{crit} for each individual point within a network, the proposed alternative procedure leads to higher quality of the detection of significant displacements at a given test significance level α.

2 DISPLACEMENT AND DISPLACEMENT ACCURACY ESTIMATION

In order to calculate the displacement and its standard deviation, the positions of identical points need to be determined in two time periods. If we have a sufficient number of stable points within a geodetic network, geodetic datum of the network is ensured well. If the calculated displacement values are significantly higher than the corresponding precision, a decision can be reached without any statistical testing. It is often hard to determine the sufficient number of stable points and detect significant displacements in geotechnical studies, for their value can often be found in the rang of their standard deviation. Statistical testing procedures are used in order to reduce the risk of credible detection of significant displacements.

Consider the position of a point P in time t and $t+\Delta t$. The displacement is calculated as follows:

$$\text{in 1D network: } d_{1D} = \Delta H = H_{t+\Delta t} - H_t, \tag{1}$$

$$\text{in 2D network: } d_{2D} = \sqrt{\Delta y^2 + \Delta x^2} = \sqrt{\left(y_{t+\Delta t} - y_t\right)^2 + \left(x_{t+\Delta t} - x_t\right)^2} \text{ in} \tag{2}$$

$$\text{in 3D network: } d_{3D} = \sqrt{\left(\Delta y^2 + \Delta x^2 + \Delta H^2\right)} = \sqrt{\left(y_{t+\Delta t} - y_t\right)^2 + \left(x_{t+\Delta t} - x_t\right)^2 + \left(H_{t+\Delta t} - H_t\right)^2}, \tag{3}$$

in which y_t, x_t, H_t and $y_{t+\Delta t}$, $x_{t+\Delta t}$, $H_{t+\Delta t}$ are adjusted coordinates of the same point in different time periods.

In order to calculate the displacement precision, we not only need to know the point coordinates, but also their variance-covariance matrix. Let's assign point P_t in the first time period with the variance-covariance matrix Σ_{P_t} and the same point $P_{t+\Delta t}$ in the second time period with the variance-covariance matrix $\Sigma_{P_{t+\Delta t}}$.

Let's assume that the coordinates of points P in time t are not correlated with the coordinates in time $t+\Delta t$. The variance-covariance matrix of the point in both time periods can be written as follows:

$$\Sigma_{P_t P_{t+\Delta t}} = \begin{bmatrix} \Sigma_{P_t} & 0 \\ 0 & \Sigma_{P_{t+\Delta t}} \end{bmatrix}, \tag{4}$$

$$\text{in 1D network: } \Sigma_{P_t P_{t+\Delta t}} = \begin{bmatrix} \sigma_{H_t}^2 & 0 \\ 0 & \sigma_{H_{t+\Delta t}}^2 \end{bmatrix}, \tag{5}$$

$$\text{in 2D network: } \Sigma_{P_t P_{t+\Delta t}} = \begin{bmatrix} \sigma_{y_t}^2 & \sigma_{y_t x_t} & 0 & 0 \\ \sigma_{y_t x_t} & \sigma_{x_t}^2 & 0 & 0 \\ 0 & 0 & \sigma_{y_{t+\Delta t}}^2 & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} \\ 0 & 0 & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} & \sigma_{x_{t+\Delta t}}^2 \end{bmatrix} \text{ in} \tag{6}$$

$$\text{in 3D network: } \Sigma_{P_t P_{t+\Delta t}} = \begin{bmatrix} \sigma_{y_t}^2 & \sigma_{y_t x_t} & \sigma_{y_t H_t} & 0 & 0 & 0 \\ \sigma_{y_t x_t} & \sigma_{x_t}^2 & \sigma_{x_t H_t} & 0 & 0 & 0 \\ \sigma_{y_t H_t} & \sigma_{x_t H_t} & \sigma_{H_t}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{y_{t+\Delta t}}^2 & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} & \sigma_{y_{t+\Delta t} H_{t+\Delta t}} \\ 0 & 0 & 0 & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} & \sigma_{x_{t+\Delta t}}^2 & \sigma_{x_{t+\Delta t} H_{t+\Delta t}} \\ 0 & 0 & 0 & \sigma_{y_{t+\Delta t} H_{t+\Delta t}} & \sigma_{x_{t+\Delta t} H_{t+\Delta t}} & \sigma_{H_{t+\Delta t}}^2 \end{bmatrix}. \tag{7}$$

Taking into account the error propagation law, the variance of point P displacement is

$$\sigma^2_{d_D} = \mathbf{J}_{d_D} \Sigma_{P_t P_{t+\Delta t}} \mathbf{J}^T_{d_D}, \tag{8}$$

in which the Jacobi matrix $\mathbf{J}_{d_D}$ equals:

$$\text{in 1D: } \mathbf{J}_{d_{1D}} = \begin{bmatrix} \frac{\partial d_{1D}}{\partial H_t} & \frac{\partial d_{1D}}{\partial H_{t+\Delta t}} \end{bmatrix} = \begin{bmatrix} -1 & 1 \end{bmatrix}, \tag{9}$$

$$\text{in 2D: } \mathbf{J}_{d_{2D}} = \begin{bmatrix} \frac{\partial d_{2D}}{\partial y_t} & \frac{\partial d_{2D}}{\partial x_t} & \frac{\partial d_{2D}}{\partial y_{t+\Delta t}} & \frac{\partial d_{2D}}{\partial x_{t+\Delta t}} \end{bmatrix} = \begin{bmatrix} -\frac{\Delta y}{d_{2D}} & -\frac{\Delta x}{d_{2D}} & \frac{\Delta y}{d_{2D}} & \frac{\Delta x}{d_{2D}} \end{bmatrix} \text{ in} \tag{10}$$

$$\text{in 3D: } \mathbf{J}_{d_{3D}} = \begin{bmatrix} \frac{\partial d_{3D}}{\partial y_t} & \frac{\partial d_{3D}}{\partial x_t} & \frac{\partial d_{3D}}{\partial H_t} & \frac{\partial d_{3D}}{\partial y_{t+\Delta t}} & \frac{\partial d_{3D}}{\partial x_{t+\Delta t}} & \frac{\partial d_{3D}}{\partial H_{t+\Delta t}} \end{bmatrix} = \begin{bmatrix} -\frac{\Delta y}{d_{3D}} & -\frac{\Delta x}{d_{3D}} & -\frac{\Delta H}{d_{3D}} & \frac{\Delta y}{d_{3D}} & \frac{\Delta x}{d_{3D}} & \frac{\Delta H}{d_{3D}} \end{bmatrix} \tag{11}$$

When we put eqs. (9), (10) and (11) into (8) the variance of point *P*'s displacement is written as follows:

$$\text{in 1D: } \sigma^2_{d_{1D}} = \sigma^2_{H_t} + \sigma^2_{H_{t+\Delta t}}, \tag{12}$$

$$\text{in 2D: } \sigma^2_{d_{2D}} = \left(\frac{\Delta y}{d_{2D}}\right)^2 \left(\sigma^2_{y_t} + \sigma^2_{y_{t+\Delta t}}\right) + \left(\frac{\Delta x}{d_{2D}}\right)^2 \left(\sigma^2_{x_t} + \sigma^2_{x_{t+\Delta t}}\right) + 2\frac{\Delta y}{d_{2D}}\frac{\Delta x}{d_{2D}}\left(\sigma_{y_t x_t} + \sigma_{y_{t+\Delta t} x_{t+\Delta t}}\right) \text{ in} \tag{13}$$

$$\text{in 3D: } \begin{aligned} \sigma^2_{d_{3D}} &= \frac{\Delta y^2}{d^2_{3D}}\left(\sigma^2_{y_t} + \sigma^2_{y_{t+\Delta t}}\right) + \frac{\Delta x^2}{d^2_{3D}}\left(\sigma^2_{x_t} + \sigma^2_{x_{t+\Delta t}}\right) + \frac{\Delta H^2}{d^2_{3D}}\left(\sigma^2_{H_t} + \sigma^2_{H_{t+\Delta t}}\right) + 2\frac{\Delta y \Delta x}{d^2_{3D}}\left(\sigma_{y_t x_t} + \sigma_{y_{t+\Delta t} x_{t+\Delta t}}\right) + \\ &+ 2\frac{\Delta y \Delta H}{d^2_{3D}}\left(\sigma_{y_t H_t} + \sigma_{y_{t+\Delta t} H_{t+\Delta t}}\right) + 2\frac{\Delta x \Delta H}{d^2_{3D}}\left(\sigma_{x_t H_t} + \sigma_{x_{t+\Delta t} H_{t+\Delta t}}\right) \end{aligned} \tag{14}$$

The variance is used for testing the statistical significance of displacements.

3 DETERMINING THE DISTRIBUTION FUNCTION OF THE TEST STATISTICS WITH SIMULATIONS

Following the adjustment of the measurements for both time periods, the displacement of points d and their accuracies σ_d are determined. The test statistic for the detection of significant displacements is written as

$$T = \frac{d}{\sigma_d}. \tag{15}$$

The test statistic can be distributed through known distribution functions (normal, student, Fischer etc.) or the distribution function can be empirically determined through simulations. It is important to accurately determine the distribution function, since the critical value T_{crit} depends on it. The critical value also depends on the significance level α and how the null hypothesis H_0 is set.

The null hypothesis has to be set for statistical testing. This determines the subset of the sample space known as the critical value approach or the null hypothesis rejection interval. An alternative hypothesis needs to be set in case the null hypothesis is rejected. The alternative hypothesis is the opposite of the null hypothesis. The test statistic is tested in relation to the null and alternative hypothesis:

H_0 : d = 0; point has not moved between the two time periods,

H_a : d ≠ 0; point has moved between the two time periods.

The test statistic T is compared to the critical value T_{crit} which is determined on the basis of the distribution function. If the test statistic is lower than the critical value at a chosen significance level α, the risk for rejecting the null hypothesis becomes too high. In this case it can be concluded that the displacement is not statistically significant. On the other hand, when the test statistic is higher than the critical value, it can be concluded that the risk for rejecting the null hypothesis is lower than the chosen significance level α. In this case the null hypothesis is rejected and followed by the conclusion that the tested displacement is statistically significant. Two cases are considered:

$T \leq T_{crit}$: null hypothesis cannot be rejected; point displacement is not statistically significant,

$T > T_{crit}$: null hypothesis must be rejected; point displacement is statistically significant.

3.1 Calculation of the critical value in a 1D network

We assume that the random measurement errors are normally distributed $\varepsilon \sim N(0, \sigma^2)$. If this is the case, then it can be safely assumed that the quantities that are linear combinations of these measurements are also normally distributed $\hat{\mathbf{x}} \sim N(\mu_{\hat{\mathbf{x}}}, \sigma^2_{\hat{\mathbf{x}}})$. Since d_{1D} is calculated as the difference of two normally distributed variables, the height difference of the same point between two time periods is also normally distributed.

3.2 Calculation of the critical value in a 2D network

Since the displacement d in 2D networks is a non-linear function of two normally distributed variables Δy and Δx it is not distributed through the normal distribution function. The displacement distribution is empirically determined through simulations (Savšek-Safić, 2002). To come up with an accurate distribution function we suggest the use of simulations with dependent random variable samples obtained through linear transformation. The procedure allows for a precise determination of the critical value at a chosen risk level (for null hypothesis rejection).

In order to generate a sample of dependent normally distributed random variables we need to generate a sample of independent normally distributed random variables upon which a linear transformation can be applied. The Box and Müller method (Box et al., 1958; Press et al., 1992) is used to generate independent normally distributed random variables. Let u_{1i} and u_{2i}, $i = 1,\ldots,n$ be two samples of independent random variables U_1 and U_2 evenly distributed on interval (0,1). A sample of two independent normally distributed random variables Z_1 and Z_2 is calculated as:

$$\mathbf{z}_i = \begin{bmatrix} z_{1i} \\ z_{2i} \end{bmatrix} = \begin{bmatrix} \sqrt{-2\ln u_{1i}}\,\sin(2\pi u_{2i}) \\ \sqrt{-2\ln u_{1i}}\,\cos(2\pi u_{2i}) \end{bmatrix},\ i = 1 \ldots n. \tag{16}$$

The linear transformation is used to convert the sample of independent normally distributed random variables into a sample of dependent normally distributed random variables

$$\mathbf{y}_i = \mathbf{U}^T \mathbf{z}_i,\ i = 1, \ldots, n \tag{17}$$

in which matrix $\mathbf{U}$ is obtained through Cholesky's decomposition of the variance covariance matrix Σ, $\Sigma = \mathbf{U}^T\mathbf{U}$

$$\mathbf{U} = \begin{bmatrix} \sigma_{\Delta y} & \dfrac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}} \\ 0 & \sigma_{\Delta x}\sqrt{1-\left(\dfrac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2} \end{bmatrix}. \tag{18}$$

When we want to generate a sample of normally distributed coordinate differences Δy_i and Δx_i we use eq. (17) and assume that the mean values $\mu_{\Delta y}$ and $\mu_{\Delta x}$ equal 0, which gives us:

$$\begin{aligned} \Delta y_i &= z_{1i}\sigma_{\Delta y} \\ \Delta x_i &= z_{1i}\frac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}} + z_{2i}\sigma_{\Delta x}\sqrt{1-\left(\frac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}. \end{aligned} \tag{19}$$

Variances and covariances are:

$$\begin{aligned} \sigma_{\Delta y} &= \sqrt{\sigma_{y_t}^2 + \sigma_{y_{t+\Delta t}}^2} \\ \sigma_{\Delta x} &= \sqrt{\sigma_{x_t}^2 + \sigma_{x_{t+\Delta t}}^2} \\ \sigma_{\Delta y \Delta x} &= \sigma_{y_t x_t} + \sigma_{y_{t+\Delta t} x_{t+\Delta t}} \end{aligned} \tag{20}$$

in which $\sigma_{y_t}^2$, $\sigma_{x_t}^2$, $\sigma_{y_t x_t}$, $\sigma_{y_{t+\Delta t}}^2$, $\sigma_{x_{t+\Delta t}}^2$, $\sigma_{y_{t+\Delta t} x_{t+\Delta t}}$ are variances and the covariances of coordinates y_t, x_t, $y_{t+\Delta t}$, $x_{t+\Delta t}$.

Since the displacement and standard deviations of point coordinates differ in each time period, the distribution function of the test statistic T is different for each point. Displacement d and its standard deviation σ_d can be calculated with the use of simulated normally distributed variables. The simulation is used for generating a sample of independent normally distributed random variables and through linear transformation we obtain dependent normally distributed random variables. In n simulations the procedure allows us to determine the empirical cumulative probability distribution of the test statistic and corresponding critical value according to the chosen significance level α for each point (Savšek-Safić et al., 2006).

3.3 Calculation of the critical value in a 3D network

Since the displacement d in 3D networks is a non-linear function of three normally distributed variables Δy, Δx and ΔH it is not distributed through the normal distribution function. The displacement distribution is empirically determined through simulations.

The distribution function is determined similarly as in 2D networks. We start off by generating a sample of independent normally distributed random variables. Let u_{1i}, u_{2i}, $i = 1,\ldots,n$ and u_{3i}, u_{4i}, $i = 1,\ldots,n$ be two pairs of samples of independent random variables U_1, U_2 and U_3, U_4 evenly distributed on the interval (0,1). A sample of three independent normally distributed random variables Z_1, Z_2 and Z_3 is calculated as follows:

$$\mathbf{z}_i = \begin{bmatrix} z_{1i} \\ z_{2i} \\ z_{3i} \end{bmatrix} = \begin{bmatrix} \sqrt{-2\ln u_{1i}}\,\sin(2\pi u_{2i}) \\ \sqrt{-2\ln u_{1i}}\,\cos(2\pi u_{2i}) \\ \sqrt{-2\ln u_{3i}}\,\sin(2\pi u_{4i}) \end{bmatrix}, \quad i = 1\ldots n \tag{21}$$

The linear transformation (17) is used to transform the sample of independent normally distributed variables into a sample of dependent normally distributed variables, for which we use the decomposed matrix **U**:

$$\mathbf{U}=\begin{bmatrix} \sigma_{\Delta y} & \dfrac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}} & \dfrac{\sigma_{\Delta y\Delta H}}{\sigma_{\Delta y}} \\ 0 & \sigma_{\Delta x}\sqrt{1-\left(\dfrac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2} & \dfrac{\sigma_{\Delta x\Delta H}\sigma_{\Delta y}^2-\sigma_{\Delta y\Delta H}\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}^2\sigma_{\Delta x}\sqrt{1-\left(\dfrac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}} \\ 0 & 0 & \sqrt{\sigma_{\Delta H}^2-\left(\dfrac{\sigma_{\Delta y\Delta H}}{\sigma_{\Delta y}}\right)^2-\left(\dfrac{\sigma_{\Delta x\Delta H}\sigma_{\Delta y}^2-\sigma_{\Delta y\Delta H}\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}^2\sigma_{\Delta x}\sqrt{1-\left(\dfrac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}}\right)^2} \end{bmatrix}. \tag{22}$$

The sample of dependent normally distributed variables Δy_i, Δx_i and ΔH_i is generated as follows:

$$\begin{aligned} \Delta y_i &= z_{1i}\sigma_{\Delta y} \\ \Delta x_i &= z_{1i}\frac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}}+z_{2i}\sigma_{\Delta x}\sqrt{1-\left(\frac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2} \\ \Delta H_i &= z_{1i}\frac{\sigma_{\Delta y\Delta H}}{\sigma_{\Delta y}}+z_{2i}\frac{\sigma_{\Delta x\Delta H}\sigma_{\Delta y}^2-\sigma_{\Delta y\Delta H}\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}^2\sigma_{\Delta x}\sqrt{1-\left(\frac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}}+z_{3i}\sqrt{\sigma_{\Delta H}^2-\left(\frac{\sigma_{\Delta y\Delta H}}{\sigma_{\Delta y}}\right)^2-\left(\frac{\sigma_{\Delta x\Delta H}\sigma_{\Delta y}^2-\sigma_{\Delta y\Delta H}\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}^2\sigma_{\Delta x}\sqrt{1-\left(\frac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}}\right)^2}. \end{aligned} \tag{23}$$

The variances and covariances of the difference between the coordinates of a specific point in two time periods are calculated as follows: for Δy and Δx through eq. (20) while for ΔH eq. (24) is used:

$$\begin{aligned} \sigma_{\Delta H} &= \sqrt{\sigma_{H_t}^2+\sigma_{H_{t+\Delta t}}^2} \\ \sigma_{\Delta y\Delta H} &= \sigma_{y_tH_t}+\sigma_{y_{t+\Delta t}H_{t+\Delta t}}, \\ \sigma_{\Delta x\Delta H} &= \sigma_{x_tH_t}+\sigma_{x_{t+\Delta t}H_{t+\Delta t}} \end{aligned} \tag{24}$$

in which $\sigma_{y_t}^2$, $\sigma_{x_t}^2$, $\sigma_{H_t}^2$, $\sigma_{y_tx_t}$, $\sigma_{y_tH_t}$, $\sigma_{x_tH_t}$, $\sigma_{y_{t+\Delta t}}^2$, $\sigma_{x_{t+\Delta t}}^2$, $\sigma_{H_{t+\Delta t}}^2$, $\sigma_{y_{t+\Delta t}x_{t+\Delta t}}$, $\sigma_{y_{t+\Delta t}H_{t+\Delta t}}$, $\sigma_{x_{t+\Delta t}H_{t+\Delta t}}$ are variances and covariances of the spatial coordinates of a point y_t, x_t, H_t, $y_{t+\Delta t}$, $x_{t+\Delta t}$, $H_{t+\Delta t}$.

In 3D and 2D networks the distribution of the test statistic T varies for each individual point.

4 RESULTS

Several control geodetic networks of different dimensions, shapes, number of reference and control points established for the purpose of deformation monitoring of dams and geological faults in the Slovene area are analyzed in the article. Stability monitoring networks typically have specific geometry (see Appendix). All observations were made using precise total station (*Leica Geosystems* TS30 R1000: $\sigma_{\text{ISO-THEO HZ,Z}} = 0.5''$ and $\sigma_{\text{ISO-EDM}} = 0.6$ mm; 1ppm) in 7 sets of angels. Observations for height determination with geometric levelling are made using precise digital level *Leica Geosystems* DNA03: $\sigma_{\text{ISO-LEV}} = 0.3$ mm/km.

The critical value T_{crit} for a normal distribution function is calculated at a selected significance level α. In practice the most common values are $\alpha = 0.10$, $\alpha = 0.05$, $\alpha = 0.01$ and $\alpha = 0.001$. Besides these values in Table 1 also shows significance levels α that provide critical values T_{crit} 3 and more.

Table 1: Critical value T_{crit} with respect to significance level α.

α	1D	2D			3D		
	T_{crit}	T^{min}_{crit}	T^{max}_{crit}	T^{mean}_{crit}	T^{min}_{crit}	T^{max}_{crit}	T^{mean}_{crit}
0.10	1.64	1.94	2.13	2.07	1.67	1.81	1.71
0.05	**1.96**	2.22	2.43	**2.36**	1.98	2.11	**2.02**
0.01	2.58	2.79	3.01	2.93	2.59	2.69	2.62
0.0027	**3.00**						
0.0081		2.86	3.08	**3.00**			
0.0084					2.97	3.07	**3.00**
0.001	3.29	3.43	3.68	3.59	3.24	3.35	3.28
0.00001	4.42	4.47	4.93	4.76	4.06	4.21	4.08

Table 1 shows that in a 1D network the critical value T_{crit} of the normal distribution function is uniquely defined at a given significance level α. When the significance value decreasing (i. e. the risk of an unjustified rejection of the null hypothesis – error type I. is reduced) the critical value rises accordingly. We can observe that for critical value $T_{crit} = 3.29$ the risk is only 0.1 %. In practice a 95 % probability is usually acceptable when deciding whether the displacement is significant or not. In this case we have to compare the test statistic T with the critical value $T_{crit} = 1.96$ and not with constants 3 or 5. If the client considers that a 5 % risk is acceptable, the given displacement can be identified as significant much sooner. This is crucial for testing as the aim is to detect significant displacement with the greatest possible probability.

In order to obtain the most probable critical value at a chosen significance level we tested several 2D and 3D geodetic networks in Slovenia with different geometries (see Tables in Appendix). Table 1 lists the minimum, maximum and average critical values T^{mean}_{crit} for test networks based on a simulated empirical distribution function for chosen significance level α=0.10; α=0.05; α=0.01 and α=0.001. Table 1 shows a significance level α, that provide an average critical values $T^{mean}_{crit} = 3$, which often occurs in practice for testing significant displacement. Level α for the critical value of 5 is not listed since its value is lower than 0.00001 and can be treated as a non-risk decision.

Table 1 indicates that when the significance value decreasing (i. e. the risk of an unjustified rejection of the null hypothesis – error type I. is reduced) the critical value rises. This can be clearly seen when we compare the test statistic T with the critical value T^{mean}_{crit} = 3.59 for 2D geodetic networks and T^{mean}_{crit} = 3.28 for 3D geodetic networks where the risk is only 0.1 %. In practice a 95 % probability is usually acceptable when deciding whether the displacement is significant or not. In such an event we have to compare the test statistic T with the critical value T^{mean}_{crit} = 2.36 for 2D geodetic networks and T^{mean}_{crit} = 2.02 for 3D geodetic networks and not with constants 3 or 5. If the client considers a 5 % risk to be acceptable, then the given displacement can be identified to be significant much sooner. This is crucial for testing as the aim is to detect significant displacement with the greatest possible probability. The client determines the acceptable risk in accordance to the consequences brought forth by a wrong decision. The aim of statistical testing is to detect significant displacement as reliably as possible. Simulation procedures can be of great help since a properly determined distribution function is of utmost importance for a critical value determination.

5 CONCLUSION

A reliable estimate of measurements, coordinates and displacements is of great importance since the deformation analysis deals with large amounts of observations in multiple time periods. Statistical testing represents an indispensable tool for evaluating parameters, identifying the adequacy of observations and mathematical models, detecting gross errors in observations, identifying the compliance of supposedly stable points during a time period and detecting statistically significant displacements of unstable points. Statistical testing procedures help us eliminate the possibility of a specific point displacement being wrongly treated as significant due to inadequate observations, a wrongly selected mathematical model or the non-compliance between the measurements in two time periods. In order to ensure reliable displacement detection it is important to select a representative test statistic when the homogeneous precision of two periods is ensured. This article discusses the common test statistic $T = d/\sigma_d$. In 2D and 3D geodetic networks the displacement is a non-linear function of variables Δy, Δx and ΔH and therefore it is not distributed through any of the analytical distribution functions.

The proposed procedure was tested on multiple geodetic networks varying in size, dimensionality (1D, 2D, 3D), geometry, number of probable stable points and number of repeated measurements (periods). In all cases the empirical distribution function was simulated and critical values were calculated for commonly used significance levels α = 0.10, α = 0.05, α = 0.01 and α = 0.001. Special attention was paid to criteria $T > 3$ and $T > 5$, known as the „rule of thumb". In the same way as the distribution function differs for each point, it also does for the critical value. In addition, the critical values that correspond to the same risk level differ for a 1D, 2D or 3D geodetic network.

The critical value is a variable so it should not be treated as a constant (3 or 5). All test cases have shown that the critical value 3 corresponds to an extremely low risk level (less than 1 %). Risk level of 1% is used when statistical tests are applied for research, risk management or in case of deformation measurement on critical infrastructure (nuclear power plants, etc.). For most geotechnical objects (bridges, dams etc.) a risk level of 5 % is sufficient. The client has greater benefits if the empirical distribution function and accurate critical values are determined according to the chosen significance/risk level. The constant risk in all network dimensions is important for the displacement evaluation as this ensures the comparability between periods.

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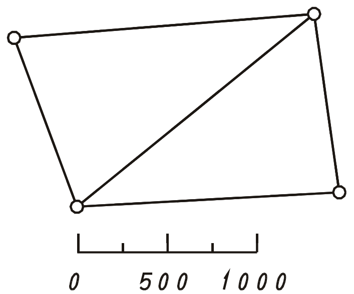
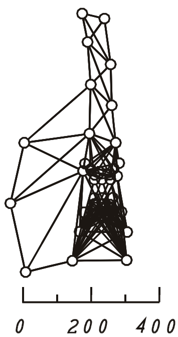
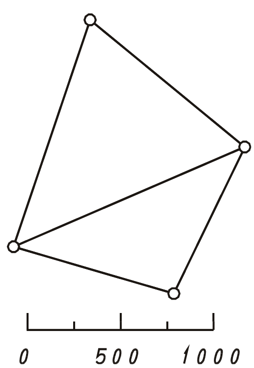
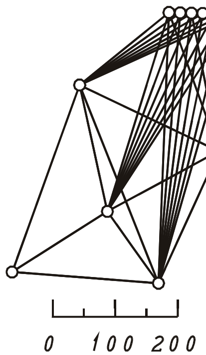
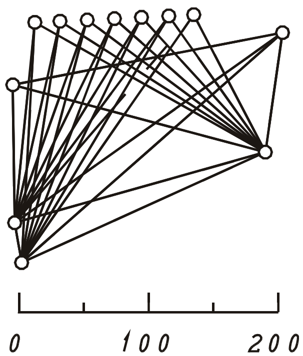
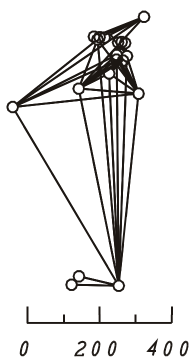
Appendix

Network shape		Number of directions	Number of distances	Num. of zenith distances	Number of unknowns	Number of points	Average σ_p (mm)	α	T_{crit}^{min}	T_{crit}^{max}	T_{crit}^{mean}
	2D	10	5		12	4	0.2	0.10	2.01	2.12	2.07
								0.05	2.30	2.42	2.63
								0.01	2.86	3.00	2.93
								0.008	2.92	3.08	3.00
								0.001	3.49	3.68	3.59
								0.00001	4.60	4.95	4.75
	3D	10	5	10	16	4	44.8	0.10	1.64	1.64	1.64
								0.05	1.96	1.96	1.96
								0.01	2.57	2.57	2.57
								0.0034	3.00	3.00	3.00
								0.001	3.23	3.23	3.23
								0.00001	4.06	4.06	4.06
	2D	93	121		99	41	0.3	0.10	1.77	2.14	2.04
								0.05	2.06	2.44	2.33
								0.01	2.65	3.02	2.91
								0.0075	2.75	3.12	3.00
								0.001	3.33	3.69	3.56
								0.00001	4.47	4.97	4.80
	3D	93	121	162	140	41	2.1	0.10	1.70	2.18	1.81
								0.05	2.01	2.46	2.10
								0.0171	2.61	3.01	2.69
								0.01	2.93	3.31	3.00
								0.001	3.27	3.64	3.34
								0.00001	4.06	4.75	4.14
	2D	10	5		12	4	0.5	0.10	2.06	2.14	2.10
								0.05	2.35	2.43	2.39
								0.01	2.93	3.02	2.98
								0.0093	2.95	3.05	3.00
								0.001	3.58	3.70	3.64
								0.00001	4.55	4.85	4.70
	3D	10	5	10	16	4	30.7	0.10	1.64	1.64	1.64
								0.05	1.96	1.96	1.96
								0.01	2.57	2.57	2.57
								0.0027	3.00	3.00	3.00
								0.001	3.23	3.23	3.23
								0.00001	4.06	4.06	4.06

Network shape		Number of directions	Number of distances	Num. of zenith distances	Number of unknowns	Number of points	Average σ_p (mm)	α	T_{crit}^{min}	T_{crit}^{max}	T_{crit}^{mean}
	2D	44	34		29	12	0.2	0.10	1.89	2.11	2.00
								0.05	2.18	2.41	2.29
								0.01	2.74	2.99	2.85
								0.0062	2.88	3.15	3.00
								0.001	3.37	3.64	3.50
								0.00001	4.51	4.90	4.67
	3D	44	34	44	41	12	3.1	0.10	1.66	1.67	1.66
								0.05	1.97	1.99	1.98
								0.01	2.58	2.59	2.58
								0.0063	3.00	3.01	3.00
								0.001	3.23	3.25	3.24
								0.00001	4.06	4.07	4.06
	2D	39	29		29	12	0.2	0.10	2.02	2.13	2.10
								0.05	2.30	2.43	2.39
								0.01	2.86	3.01	2.97
								0.0090	2.90	3.04	3.00
								0.001	3.50	3.68	3.62
								0.00001	4.45	4.95	4.82
	3D	39	28	36	41	12	1.5	0.10	1.71	1.94	1.79
								0.05	2.02	2.22	2.09
								0.01	2.62	2.77	2.67
								0.0023	2.94	3.08	3.00
								0.001	3.27	3.41	3.33
								0.00001	4.07	4.21	4.09
	2D	76	45		51	19	0.2	0.10	1.87	2.14	2.08
								0.05	2.16	2.44	2.37
								0.01	2.71	3.03	2.95
								0.0085	2.76	3.08	3.00
								0.001	3.34	3.70	3.61
								0.00001	4.25	4.97	4.80
	3D	76	45	76	70	19	2.4	0.10	1.66	1.76	1.72
								0.05	1.97	2.06	2.02
								0.0188	2.58	2.65	2.62
								0.01	2.96	3.04	3.00
								0.001	3.23	3.31	3.29
								0.00001	4.03	4.10	4.07

ALTERNATIVNA METODA TESTIRANJA PREMIKOV V GEODETSKI MREŽI

OSNOVNE INFORMACIJE O ČLANKU:

GLEJ STRAN 387

1 UVOD

Strokovnjaki, ki delujejo na področju geoznanosti (gradbeniki, geologi, rudarji in drugi), v svojih raziskavah opredelijo pričakovane premike. Geodeti na podlagi terenskih meritev določimo dejanske premike kot na primer Marjetič et al. (2012), Kregar et al. (2015), in natančnost določitve premikov z uporabo statističnih metod kot na primer Kregar et al. (2012), Urbančič et al. (2016). Nalogo lahko tudi obrnemo. Glede na natančnost določitve premikov, ki je posledica merjenja in izračunov, lahko izračunamo mejo, do katere lahko trdimo, da točka miruje ali se je značilno premaknila. V literaturi je opisanih nekaj pristopov. V pregledu literature ohranjamo oznake tako, kot jih zapisujejo posamezni avtorji.

Klasični pristop ocenjevanja, ali je premik statistično značilen ali ne, je uporabljen in opisan v številnih člankih (Caspary, 1987; Dognan et al., 2013; Heck et al., 1977; Koch, 1999; Kuang, 1996; Niemer, 1985; Pelzer, 1971; Savšek-Safić et al., 2003; Sütti in Török, 1996). Kritična vrednost pri izbrani stopnji značilnosti α se primerja s kvocientom kvadratne forme, ki se izračuna kot produkt vektorja premika točk s pripadajočo matriko kofaktorjev $T = \mathbf{d}^T \Sigma^{-1}_{\mathbf{dd}} \mathbf{d}$, ta se porazdeljuje po porazdelitvi χ^2_α (Koch, 1999).

Statistično značilne premike lahko testiramo s klasičnimi postopki deformacijske analize (angl. *CDA – conventional deformation analysis*) ali z robustnimi metodami (angl. *IWST – iterative weighted similarity transformation*). Izbira metode je običajno odvisna od vrste deformacij, ki jih raziskujemo, in od vrste kontrolne mreže, ki jo vzpostavimo za obravnavani objekt. CDA-metode temeljijo na oceni najmanjših kvadratov (LSE), kot sta Hannover in Karlsruhe, kjer uporabimo globalni kongruenčni test med dvema terminskima izmerama (Pelzer, 1971; Niemeier, 1981). IWST-metode zagotavljajo dobro oceno vektorja premikov za posamezno točko v postopku robustne S-transformacije (Chen et al., 1990). V literaturi je predstavljena alternativna M-ocena za določitev vektorja premikov neposredno iz razlik »surovih« neizravnanih meritev (Nowel, 2015; Nowel, 2016). Metoda je še posebej učinkovita, kadar se pričakujejo mali premiki, ki le malo presežejo merske pogreške. Hekimoglu in Koch (2000) predlagata zelo dobro empirično mero za učinkovito testiranje grobih pogreškov in oceno robustnih metod, imenovano povprečna ocena uspešnosti (angl. *MSR – mean success rate*). Izračunamo jo na podlagi velikega števila simuliranih meritev po metodi Monte Carlo kot razmerje med številom meritev, pri katerih odkrijemo grobe pogreške, in številom vseh meritev. Predlagana mera MSR je lahko dobra alternativa statističnim meram, kot so globalne in lokalne mere notranje zanesljivosti v tradicionalnih pristopih deformacijske analize.

Rueger (1999) obravnava zahtevano natančnost določitve premikov podobno kot Welsch et al. (2000) in Pelzer et al. (1987). Če je ocenjena velikost minimalnega premika d_y, potem lahko zahtevano natančnost meritev s_y ocenimo s približno enačbo $s_y \leq d/5$. V drugih člankih avtorji predpostavijo, da je premik značilen, če je kvocient med premikom in standardno deviacijo premika večji kot 3 (Klein in Heunecke, 2006).

V ameriški agenciji za vojno inženirstvo USACE (USACE Army, 2002) priporočajo, naj bo standardna deviacija meritev ($\sigma_{39\%}$) za določitev deformacij devetkrat manjša od največje pričakovane vrednosti deformacij. Če uporabimo 95 % stopnjo zaupanja, pa mora biti standardna deviacija meritev ($\sigma_{95\%}$) štirikrat manjša od največje pričakovane vrednosti deformacij.

Hekimoglu et al. (2010) izračuna radij pripadajočega kroga premikov r iz elementov pripadajoče podmatrike matrike kofaktorjev $\mathbf{Q}_{\mathbf{dd}}$ in izbrane stopnje značilnosti α porazdelitve χ^2_2 za 2 prostostni stopnji $\chi^2_{2,\alpha}$. Ob izbrani stopnji značilnosti α ugotavlja, koliko odstotkov točk se je premaknilo v intervalu med r in $2r$ oziroma med r in $3r$.

Ramos et al. (2012) so razvili program za 3D-premike oljk, ki jih povzroča erozija terena. Premike so analizirali ločeno za horizontalno in vertikalno komponento. Premike v ravnini so obravnavali kot značilne, kadar se 99 % elipsi pogreškov iz dveh zaporednih izmer ne prekrivata. V višinskem smislu so premik označili kot značilen, kadar je večji od intervala natančnosti $\sqrt{Error\_C1^2_{99\ \%} + Error\_C2^2_{99\ \%}}$.

Do sedaj opravljen pregled literature obravnava premike pretežno v 2D-ravnini. Berber (2006) in Berber et al. (2009) izračunajo mejno vrednost 3D-elipsoida zaupanja z enačbo $\delta_i = \sqrt{\sigma^2_{a_{95_i}} + \sigma^2_{b_{95_i}} + \sigma^2_{h_{95_i}}}$, kjer so posamezne polosi 95 % elipsoida zaupanja in vertikalnega intervala izračunane kot $\sigma_{a,b,h95_i} = 2{,}795\sigma_{a,b,\ h_i}$. Standardni 3D-elipsoid ima pri $\chi^2_{3;1-\alpha}$ stopnjo zaupanja (1-α) približno 20 %. Da dosežemo 95 % verjetnost, je treba osnovno enačbo standardnega 3D-elipsoida pomnožiti s faktorjem 2,795 (Staudinger, 1999).

Premiki in deformacije nastanejo tako na umetnih objektih, kot so jezovi, nasipi, mostovi, kot tudi v njihovi okolici, na primer v dolinah jezov, na obrežjih umetnih akumulacij, pa tudi na naravnih območjih, kot so plazovi, ob tektonskih prelomnicah, na barjanskih tleh. V splošnem lahko vzroke za nastanek premikov in deformacij pripisujemo delovanju zunanjih vplivov (spremembi temperature, vetru, tektonskim in seizmičnim vplivom), mehaničnim lastnostim gradbenega materiala in konstrukcijskih elementov ter nezadostnemu upoštevanju geološke sestave, mehanskih lastnosti ter hidroloških pogojev tal pri projektiranju objekta. Določitev premikov in deformacij naravnih in umetnih objektov je ena zahtevnejših nalog geodetske stroke.

Pri presoji statistično značilnih premikov navadno uporabimo indirektno določene količine, na primer premik in pripadajočo standardno deviacijo, ki jih z zapisom testne statistike $T = d/\sigma_d$ testiramo s postopki statističnega testiranja hipotez. Ker testno statistiko T primerjamo s kritično vrednostjo T_{crit}, ki jo določimo na podlagi ustrezne porazdelitvene funkcije in izbrane stopnje značilnosti testa α, je zelo pomembno, da pravilno določimo porazdelitveno funkcijo, po kateri se testna statistika porazdeljuje. Statistični test predpisuje pravilo za določitev neke podmnožice prostora vzorcev, ki jo imenujemo *kritično območje preizkusa* ali *območje zavrnitve* ničelne hipoteze. V praksi se pri presoji o značilnih premikih pogosto uporabi približni kriterij $T > 3$ ali $T > 5$, ki pa je pogosto preohlapen, saj je s postopki simulacij mogoče natančneje izračunati kritično vrednost T_{crit} porazdelitvene funkcije.

V članku obravnavamo posebnosti pri izračunu premika in pripadajoče standardne deviacije v 1D-, 2D- in 3D-geodetskih mrežah. Zapišemo postopek za določitev porazdelitvene funkcije za testno statistiko $T = d/\sigma_d$ glede na dimenzijo mreže. V ravninski in prostorski mreži se izračunana testna statistika ne porazdeljuje po nobeni od znanih porazdelitvenih funkcij, zato pripadajočo porazdelitveno funkcijo določimo empirično s simulacijami. S simulacijami lahko določimo kritično vrednost T_{crit} porazdelitvene funkcije za vsako točko v mreži, zato lahko predlagani alternativni postopek zagotovi večjo občutljivost pri ugotavljanju značilnih premikov pri izbrani stopnji zaupanja α.

2 OCENA PREMIKOV IN NJIHOVE NATANČNOSTI

Pogoj za izračun premika in pripadajoče standardne deviacije premika je, da so identične točke izmerjene najmanj v dveh terminskih izmerah. Če obstaja dovolj stabilnih točk, ki zagotavljajo kakovosten geodetski datum v obeh izmerah, izračunani premiki pa so značilno večji od standardne deviacije, se za premik lahko opredelijo iz razlike koordinat v dveh terminskih izmerah in dodatno statistično testiranje ni potrebno. V geotehničnih raziskavah je pogosto težavno določiti dovolj stabilnih točk, kakor tudi značilne premike, ki so pogosto le malo večji od standardne deviacije premika. Da bi zmanjšali tveganje pri verodostojni obravnavi značilnih premikov, uporabimo postopke statističnega testiranja, ki nam olajšajo odločitve.

Predpostavimo, da obravnavamo položaj točke P v času t in $t + \Delta t$. Premik točke izračunamo:

$$\text{v 1D-mreži: } d_{1D} = \Delta \mathrm{H} = H_{t+\Delta t} - H_t, \tag{1}$$

$$\text{v 2D-mreži: } d_{2D} = \sqrt{\Delta y^2 + \Delta x^2} = \sqrt{\left(y_{t+\Delta t} - y_t\right)^2 + \left(x_{t+\Delta t} - x_t\right)^2} \text{ in} \tag{2}$$

$$\text{v 3D-mreži: } d_{3D} = \sqrt{\left(\Delta y^2 + \Delta x^2 + \Delta H^2\right)} = \sqrt{\left(y_{t+\Delta t} - y_t\right)^2 + \left(x_{t+\Delta t} - x_t\right)^2 + \left(H_{t+\Delta t} - H_t\right)^2}, \tag{3}$$

kjer so y_t, x_t, H_t in $y_{t+\Delta t}$, $x_{t+\Delta t}$, $H_{t+\Delta t}$ izravnane koordinate ene točke v dveh terminskih izmerah.

Da bi lahko izračunali natančnost premika točke, moramo poleg koordinat točke poznati tudi kovariančno matriko koordinat točke za posamezno terminsko izmero. Naj ima točka P_t v prvi izmeri pripadajočo variančno-kovariančno matriko Σ_{P_t} in identična točka $P_{t+\Delta t}$ v drugi izmeri pripadajočo variančno-kovariančno matriko $\Sigma_{P_{t+\Delta t}}$. Predpostavimo, da so koordinate točke P v času t nekorelirane s koordinatami v času $t+\Delta t$. Variančno-kovariančno matriko identične točke v dveh časovno neodvisnih izmerah lahko zapišemo:

$$\Sigma_{P_t P_{t+\Delta t}} = \begin{bmatrix} \Sigma_{P_t} & 0 \\ 0 & \Sigma_{P_{t+\Delta t}} \end{bmatrix}, \tag{4}$$

$$\text{v 1D-mreži: } \Sigma_{P_t P_{t+\Delta t}} = \begin{bmatrix} \sigma^2_{H_t} & 0 \\ 0 & \sigma^2_{H_{t+\Delta t}} \end{bmatrix}, \tag{5}$$

$$\text{v 2D-mreži: } \Sigma_{P_t P_{t+\Delta t}} = \begin{bmatrix} \sigma^2_{y_t} & \sigma_{y_t x_t} & 0 & 0 \\ \sigma_{y_t x_t} & \sigma^2_{x_t} & 0 & 0 \\ 0 & 0 & \sigma^2_{y_{t+\Delta t}} & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} \\ 0 & 0 & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} & \sigma^2_{x_{t+\Delta t}} \end{bmatrix} \text{ in} \tag{6}$$

$$\text{v 3D-mreži: } \Sigma_{P_t P_{t+\Delta t}} = \begin{bmatrix} \sigma^2_{y_t} & \sigma_{y_t x_t} & \sigma_{y_t H_t} & 0 & 0 & 0 \\ \sigma_{y_t x_t} & \sigma^2_{x_t} & \sigma_{x_t H_t} & 0 & 0 & 0 \\ \sigma_{y_t H_t} & \sigma_{x_t H_t} & \sigma^2_{H_t} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{y_{t+\Delta t}} & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} & \sigma_{y_{t+\Delta t} H_{t+\Delta t}} \\ 0 & 0 & 0 & \sigma_{y_{t+\Delta t} x_{t+\Delta t}} & \sigma^2_{x_{t+\Delta t}} & \sigma_{x_{t+\Delta t} H_{t+\Delta t}} \\ 0 & 0 & 0 & \sigma_{y_{t+\Delta t} H_{t+\Delta t}} & \sigma_{x_{t+\Delta t} H_{t+\Delta t}} & \sigma^2_{H_{t+\Delta t}} \end{bmatrix}. \tag{7}$$

Po zakonu o prenosu varianc in kovarianc določimo natančnost premika

$$\sigma_{d_D}^2 = \mathbf{J}_{d_D} \Sigma_{P_t P_{t+\Delta t}} \mathbf{J}_{d_D}^T, \tag{8}$$

kjer je Jacobijeva matrika $\mathbf{J}_{d_D}$ enaka

$$\text{v 1D: } \mathbf{J}_{d_{1D}} = \begin{bmatrix} \frac{\partial d_{1D}}{\partial H_t} & \frac{\partial d_{1D}}{\partial H_{t+\Delta t}} \end{bmatrix} = \begin{bmatrix} -1 & 1 \end{bmatrix}, \tag{9}$$

$$\text{v 2D: } \mathbf{J}_{d_{2D}} = \begin{bmatrix} \frac{\partial d_{2D}}{\partial y_t} & \frac{\partial d_{2D}}{\partial x_t} & \frac{\partial d_{2D}}{\partial y_{t+\Delta t}} & \frac{\partial d_{2D}}{\partial x_{t+\Delta t}} \end{bmatrix} = \begin{bmatrix} -\frac{\Delta y}{d_{2D}} & -\frac{\Delta x}{d_{2D}} & \frac{\Delta y}{d_{2D}} & \frac{\Delta x}{d_{2D}} \end{bmatrix} \text{ in} \tag{10}$$

$$\text{v 3D: } \mathbf{J}_{d_{3D}} = \begin{bmatrix} \frac{\partial d_{3D}}{\partial y_t} & \frac{\partial d_{3D}}{\partial x_t} & \frac{\partial d_{3D}}{\partial H_t} & \frac{\partial d_{3D}}{\partial y_{t+\Delta t}} & \frac{\partial d_{3D}}{\partial x_{t+\Delta t}} & \frac{\partial d_{3D}}{\partial H_{t+\Delta t}} \end{bmatrix} = \begin{bmatrix} -\frac{\Delta y}{d_{3D}} & -\frac{\Delta x}{d_{3D}} & -\frac{\Delta H}{d_{3D}} & \frac{\Delta y}{d_{3D}} & \frac{\Delta x}{d_{3D}} & \frac{\Delta H}{d_{3D}} \end{bmatrix} \tag{11}$$

Če vstavimo enačbe (9), (10) in (11) v enačbo (8), varianco premika točke *P* lahko zapišemo

$$\text{v 1D: } \sigma_{d_{1D}}^2 = \sigma_{H_t}^2 + \sigma_{H_{t+\Delta t}}^2, \tag{12}$$

$$\text{v 2D: } \sigma_{d_{2D}}^2 = \left(\frac{\Delta y}{d_{2D}}\right)^2 \left(\sigma_{y_t}^2 + \sigma_{y_{t+\Delta t}}^2\right) + \left(\frac{\Delta x}{d_{2D}}\right)^2 \left(\sigma_{x_t}^2 + \sigma_{x_{t+\Delta t}}^2\right) + 2\frac{\Delta y}{d_{2D}}\frac{\Delta x}{d_{2D}}\left(\sigma_{y_t x_t} + \sigma_{y_{t+\Delta t} x_{t+\Delta t}}\right) \text{ in} \tag{13}$$

$$\text{v 3D: } \begin{aligned} \sigma_{d_{3D}}^2 = {} & \frac{\Delta y^2}{d_{3D}^2}\left(\sigma_{y_t}^2 + \sigma_{y_{t+\Delta t}}^2\right) + \frac{\Delta x^2}{d_{3D}^2}\left(\sigma_{x_t}^2 + \sigma_{x_{t+\Delta t}}^2\right) + \frac{\Delta H^2}{d_{3D}^2}\left(\sigma_{H_t}^2 + \sigma_{H_{t+\Delta t}}^2\right) + 2\frac{\Delta y \Delta x}{d_{3D}^2}\left(\sigma_{y_t x_t} + \sigma_{y_{t+\Delta t} x_{t+\Delta t}}\right) + \\ & + 2\frac{\Delta y \Delta H}{d_{3D}^2}\left(\sigma_{y_t H_t} + \sigma_{y_{t+\Delta t} H_{t+\Delta t}}\right) + 2\frac{\Delta x \Delta H}{d_{3D}^2}\left(\sigma_{x_t H_t} + \sigma_{x_{t+\Delta t} H_{t+\Delta t}}\right) \end{aligned} \tag{14}$$

Varianco premika uporabimo za testiranje statistično značilnih premikov.

3 DOLOČITEV PORAZDELITVENE FUNKCIJE S SIMULACIJAMI

Po izravnavi najmanj dveh terminskih izmer je mogoče določiti premik točke d in standardno deviacijo premika σ_d. Zapišimo ustrezno testno statistiko, s katero skušamo zaznati značilne spremembe točk v mreži, ki nas zanimajo:

$$T = \frac{d}{\sigma_d}. \tag{15}$$

Testna statistika se lahko porazdeljuje po znanih porazdelitvenih funkcijah (normalna, Studentova, Fisherjeva idr.) ali pa je porazdelitveno funkcijo treba določiti analitično ali empirično s simulacijami. Natančna določitev porazdelitvene funkcije, po kateri se porazdeljuje testna statistika T, je zelo pomembna, saj se glede na porazdelitveno funkcijo izračuna kritična vrednost T_{crit}. Za izračun kritične vrednosti T_{crit} je pomembno tudi, kakšno vrednost stopnje značilnosti testa α izberemo in kako postavimo ničelno hipotezo H_0.

V postopku testiranja hipotez je treba postaviti ničelno hipotezo, s katero se predpisuje pravilo za določitev podmnožice prostora vzorcev, ki jo imenujemo kritično območje preizkusa ali območje zavrnitve ničelne hipoteze. V primeru zavrnitve ničelne hipoteze zapišemo ustrezno alternativno hipotezo. Testno statistiko testiramo glede na postavljeno ničelno H_0 in alternativno hipotezo H_a:

$H_0 : d = 0$; točka v obdobju dveh terminskih izmer miruje,

$H_a : d \neq 0$; točka se je v obdobju dveh terminskih izmer značilno premaknila.

Testno statistiko T primerjamo glede na kritično vrednost T_{crit}, ki jo izračunamo na podlagi porazdelitvene funkcije. Ko je testna statistika manjša od kritične vrednosti ob izbrani stopnji značilnosti testa α, je tveganje za zavrnitev ničelne hipoteze preveliko. V tem primeru ugotovimo, da premik ni statistično značilen. Če je vrednost testne statistike večja od kritične vrednosti porazdelitvene funkcije, pa ugotovimo, da je tveganje za zavrnitev ničelne hipoteze manjše od izbrane stopnje značilnosti testa α. Zato upravičeno zavrnemo ničelno hipotezo in tako potrdimo, da je obravnavani premik statistično značilen. Obravnavamo torej dva primera:

$T \leq T_{crit}$: ne zavrnemo ničelne hipoteze; premik točke ni statistično značilen,

$T > T_{crit}$: zavrnemo ničelno hipotezo; premik točke je statistično značilen.

3.1 Izračun kritične vrednosti v 1D-mreži

Če predpostavimo, da so pogreški opazovanj normalno porazdeljeni $\varepsilon \sim N(0, \sigma^2)$, se enako porazdeljujejo tudi količine, ki so linearne funkcije opazovanj $\hat{\mathbf{x}} \sim N(\mu_{\hat{\mathbf{x}}}, \sigma^2_{\hat{\mathbf{x}}})$. Ker d_{1D} izračunamo kot razliko dveh normalno porazdeljenih slučajnih spremenljivk (glej poglavje 2), je tudi razlika višin identične točke med dvema terminskima izmerama normalno porazdeljena količina.

3.2 Izračun kritične vrednosti v 2D-mreži

Premik d je v 2D-mrežah nelinearna funkcija normalno porazdeljenih spremenljivk Δy in Δx, zato se ne porazdeljuje po normalni porazdelitveni funkciji. Empirično porazdelitveno funkcijo določimo s simulacijami (Savšek-Safić, 2002). Za določitev natančne porazdelitvene funkcije predlagamo uporabo simulacij za pridobitev vzorca odvisnih normalno porazdeljenih slučajnih spremenljivk s pomočjo linearne transformacije. Postopek omogoča natančno določitev kritične vrednosti pri izbrani stopnji tveganja (za zavrnitev ničelne hipoteze).

Osnovna ideja za generiranje vzorca *odvisnih* normalno porazdeljenih slučajnih spremenljivk je, da najprej generiramo vzorec neodvisnih normalno porazdeljenih spremenljivk, potem pa uporabimo linearno transformacijo za pridobitev vzorca odvisnih slučajnih spremenljivk. Za generiranje vzorca normalno porazdeljene slučajne spremenljivke uporabimo metodo Box in Müller (Box et al., 1958; Press et al., 1992). Naj bosta u_{1i} in u_{2i}, $i = 1, ..., n$ dva vzorca slučajnih spremenljivk U_1 in U_2, ki sta neodvisni in porazdeljeni enakomerno na intervalu (0,1). Vzorec dveh neodvisnih standardizirano normalno porazdeljenih slučajnih spremenljivk Z_1 in Z_2 izračunamo

$$\mathbf{z}_i = \begin{bmatrix} z_{1i} \\ z_{2i} \end{bmatrix} = \begin{bmatrix} \sqrt{-2\ln u_{1i}}\,\sin\left(2\pi u_{2i}\right) \\ \sqrt{-2\ln u_{1i}}\,\cos\left(2\pi u_{2i}\right) \end{bmatrix},\ i = 1 \ldots n\,. \tag{16}$$

Za pretvorbo vzorca neodvisnih normalno porazdeljenih spremenljivk v vzorec odvisnih normalno porazdeljenih slučajnih spremenljivk uporabimo linearno transformacijo

$$\mathbf{y}_i = \mathbf{U}^T \mathbf{z}_i,\ i = 1, ..., n, \tag{17}$$

kjer matriko **U** dobimo s Cholesky razcepom variančno-kovariančne matrike Σ, $\Sigma = \mathbf{U}^T\mathbf{U}$

$$\mathbf{U} = \begin{bmatrix} \sigma_{\Delta y} & \dfrac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}} \\ 0 & \sigma_{\Delta x}\sqrt{1-\left(\dfrac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2} \end{bmatrix}. \tag{18}$$

Če želimo generirati vzorec normalno porazdeljenih koordinatnih razlik Δy_i in Δx_i, izračunamo po (17) in predpostavimo, da sta srednji vrednosti $\mu_{\Delta y}$ in $\mu_{\Delta x}$ enaki 0, dobimo:

$$\begin{aligned} \Delta y_i &= z_{1i}\sigma_{\Delta y} \\ \Delta x_i &= z_{1i}\frac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}} + z_{2i}\sigma_{\Delta x}\sqrt{1-\left(\frac{\sigma_{\Delta y \Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}. \end{aligned} \tag{19}$$

Variance in kovariance izračunamo

$$\begin{aligned} \sigma_{\Delta y} &= \sqrt{\sigma^2_{y_t} + \sigma^2_{y_{t+\Delta t}}} \\ \sigma_{\Delta x} &= \sqrt{\sigma^2_{x_t} + \sigma^2_{x_{t+\Delta t}}} \quad , \\ \sigma_{\Delta y \Delta x} &= \sigma_{y_t x_t} + \sigma_{y_{t+\Delta t} x_{t+\Delta t}} \end{aligned} \tag{20}$$

kjer so $\sigma^2_{y_t}$, $\sigma^2_{x_t}$, $\sigma_{y_t x_t}$, $\sigma^2_{y_{t+\Delta t}}$, $\sigma^2_{x_{t+\Delta t}}$, $\sigma_{y_{t+\Delta t} x_{t+\Delta t}}$ variance in kovariance ravninskih koordinat točke y_t, x_t, $y_{t+\Delta t}$, $x_{t+\Delta t}$.

Porazdelitvena funkcija testne statistike T je za vsako točko drugačne oblike, saj sta premik in standardna deviacija koordinat točk v posamezni terminski izmeri za različne točke različna. Z uporabo simuliranih normalno porazdeljenih spremenljivk izračunamo premik d in standardno deviacijo premika σ_d. Simulacije uporabimo za generiranje vzorca neodvisnih normalno porazdeljenih slučajnih spremenljivk in z linearno transformacijo pridobimo odvisne normalno porazdeljene slučajne spremenljivke. V n simulacijah nam postopek omogoča določitev empirične kumulativne verjetnostne porazdelitvene funkcije testne statistike in izračun pripadajoče kritične vrednosti glede na izbrano stopnjo značilnosti testa α za vsako posamezno točko (Savšek-Safić et al., 2006).

3.3 Izračun kritične vrednosti v 3D-mreži

Premik d je tudi v 3D-mrežah nelinearna funkcija normalno porazdeljenih spremenljivk Δy, Δx_i in ΔH, zato se ne porazdeljuje po normalni porazdelitveni funkciji in moramo empirično porazdelitveno funkcijo določiti s simulacijami.

Porazdelitveno funkcijo določimo podobno kot v 2D-mrežah, tako da najprej generiramo vzorca neodvisnih normalno porazdeljenih slučajnih spremenljivk. Naj bosta u_{1i}, u_{2i}, $i = 1,\ldots,n$ in u_{3i}, u_{4i}, $i = 1,\ldots,n$ dva para vzorcev slučajnih spremenljivk U_1, U_2 in U_3, U_4, ki sta neodvisna in porazdeljena enakomerno na intervalu (0,1). Vzorec treh neodvisnih standardizirano normalno porazdeljenih slučajnih spremenljivk Z_1, Z_2 in Z_3 izračunamo po enačbi:

$$\mathbf{z}_i = \begin{bmatrix} z_{1i} \\ z_{2i} \\ z_{3i} \end{bmatrix} = \begin{bmatrix} \sqrt{-2\ln u_{1i}}\,\sin(2\pi u_{2i}) \\ \sqrt{-2\ln u_{1i}}\,\cos(2\pi u_{2i}) \\ \sqrt{-2\ln u_{3i}}\,\sin(2\pi u_{4i}) \end{bmatrix},\ \ i = 1 \ldots n\cdot \tag{21}$$

Nato uporabimo linearno transformacijo (17) za pretvorbo vzorca neodvisnih normalno porazdeljenih spremenljivk v vzorec odvisnih normalno porazdeljenih slučajnih spremenljivk, kjer se matrika **U** izračunana kot:

$$\mathbf{U} = \begin{bmatrix} \sigma_{\Delta y} & \dfrac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}} & \dfrac{\sigma_{\Delta y\Delta H}}{\sigma_{\Delta y}} \\ 0 & \sigma_{\Delta x}\sqrt{1-\left(\dfrac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2} & \dfrac{\sigma_{\Delta x\Delta H}\sigma_{\Delta y}^2 - \sigma_{\Delta y\Delta H}\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}^2\sigma_{\Delta x}\sqrt{1-\left(\dfrac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}} \\ 0 & 0 & \sqrt{\sigma_{\Delta H}^2 - \left(\dfrac{\sigma_{\Delta y\Delta H}}{\sigma_{\Delta y}}\right)^2 - \left(\dfrac{\sigma_{\Delta x\Delta H}\sigma_{\Delta y}^2 - \sigma_{\Delta y\Delta H}\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}^2\sigma_{\Delta x}\sqrt{1-\left(\dfrac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}}\right)^2} \end{bmatrix}. \tag{22}$$

Generiranje vzorca odvisnih normalno porazdeljenih Δy_i, Δx_i in ΔH_i izračunamo po naslednjih enačbah:

$$\begin{aligned} \Delta y_i &= z_{1i}\sigma_{\Delta y} \\ \Delta x_i &= z_{1i}\frac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}} + z_{2i}\sigma_{\Delta x}\sqrt{1-\left(\frac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2} \\ \Delta H_i &= z_{1i}\frac{\sigma_{\Delta y\Delta H}}{\sigma_{\Delta y}} + z_{2i}\frac{\sigma_{\Delta x\Delta H}\sigma_{\Delta y}^2 - \sigma_{\Delta y\Delta H}\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}^2\sigma_{\Delta x}\sqrt{1-\left(\frac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}} + z_{3i}\sqrt{\sigma_{\Delta H}^2 - \left(\frac{\sigma_{\Delta y\Delta H}}{\sigma_{\Delta y}}\right)^2 - \left(\frac{\sigma_{\Delta x\Delta H}\sigma_{\Delta y}^2 - \sigma_{\Delta y\Delta H}\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}^2\sigma_{\Delta x}\sqrt{1-\left(\frac{\sigma_{\Delta y\Delta x}}{\sigma_{\Delta y}\sigma_{\Delta x}}\right)^2}}\right)^2} \end{aligned} \tag{23}$$

Variance in kovariance razlik koordinat točke v dveh terminskih izmerah Δy in Δx izračunamo po enačbi (20), variance in kovariance v 3D-mreži pa po naslednjih enačbah:

$$\begin{aligned} \sigma_{\Delta H} &= \sqrt{\sigma_{H_t}^2 + \sigma_{H_{t+\Delta t}}^2} \\ \sigma_{\Delta y\Delta H} &= \sigma_{y_t H_t} + \sigma_{y_{t+\Delta t}H_{t+\Delta t}}, \\ \sigma_{\Delta x\Delta H} &= \sigma_{x_t H_t} + \sigma_{x_{t+\Delta t}H_{t+\Delta t}} \end{aligned} \tag{24}$$

kjer so $\sigma^2_{y_t}$, $\sigma^2_{x_t}$, $\sigma^2_{H_t}$, $\sigma_{y_t x_t}$, $\sigma_{y_t H_t}$, $\sigma_{x_t H_t}$, $\sigma^2_{y_{t+\Delta t}}$, $\sigma^2_{x_{t+\Delta t}}$, $\sigma^2_{H_{t+\Delta t}}$, $\sigma_{y_{t+\Delta t} x_{t+\Delta t}}$, $\sigma_{y_{t+\Delta t} H_{t+\Delta t}}$, $\sigma_{x_{t+\Delta t} H_{t+\Delta t}}$ variance in kovariance prostorskih koordinat točke y_t, x_t, H_t, $y_{t+\Delta t}$, $x_{t+\Delta t}$, $H_{t+\Delta t}$.

V 3D- in 2D-mrežah je porazdelitvena funkcija testne statistike T različna za vsako točko.

4 REZULTATI

Na območju Slovenije so vzpostavljene številne geodetske kontrolne mreže za spremljanje premikov pregradnih objektov in geoloških prelomov. V članku analiziramo nekatere kontrolne mreže različnih dimenzij, geometrije in različnega števila referenčnih in kontrolnih točk (glej prilogo). Meritve so bile izvedene s preciznim elektronskim tahimetrom (*Leica Geosystems* TS30 R1000: $\sigma_{\text{ISO-THEO HZ,Z}}$ = 0,5” in $\sigma_{\text{ISO-EDM}}$ = 0,6 mm; 1ppm) v 7 girusih. Za določitev višin z geometričnim nivelmanom smo uporabili precizni digitalni nivelir *Leica Geosystems* DNA03: $\sigma_{\text{ISO-LEV}}$ = 0,3 mm/km.

Za normalno porazdelitev določimo kritično vrednost T_{crit} glede na izbrano stopnjo značilnosti testa α. V praksi najpogosteje uporabljamo vrednosti α = 0,10; α = 0,05; α = 0,01 in α = 0,001. Poleg kritičnih vrednosti za te α v preglednici 1 podajamo še vednosti α za T_{crit} 3 in več.

Preglednica 1: Kritična vrednost T_{crit} glede na izbrano stopnjo zaupanja α.

α	1D	2D			3D		
	T_{crit}	T^{min}_{crit}	T^{max}_{crit}	T^{mean}_{crit}	T^{min}_{crit}	T^{max}_{crit}	T^{mean}_{crit}
0,10	1,64	1,94	2,13	2,07	1,67	1,81	1,71
0,05	**1,96**	2,22	2,43	**2,36**	1,98	2,11	**2,02**
0,01	2,58	2,79	3,01	2,93	2,59	2,69	2,62
0,0027	**3,00**						
0,0081		2,86	3,08	**3,00**			
0,0084					2,97	3,07	**3,00**
0,001	3,29	3,43	3,68	3,59	3,24	3,35	3,28
0,00001	4,42	4,47	4,93	4,76	4,06	4,21	4,08

Kot vidimo iz preglednice 1, je v 1D-mreži pri izbrani stopnji značilnosti testa α kritična vrednost normalne porazdelitvene funkcije T_{crit} enolično določena. Z zmanjševanjem stopnje značilnosti testa oziroma tveganja za neupravičeno zavrnitev ničelne hipoteze (napaka I. vrste) se kritična vrednost ustrezno povečuje. Iz preglednice 1 je razvidno, da je pri kritični vrednosti T_{crit} = 3,29 tveganje minimalno in znaša 0,1 %. V praksi pogosto navajajo 95-odstotno verjetnost kot sprejemljivo verjetnost pri odločitvi, ali neki premik obravnavamo kot statistično značilen ali ne. V tem primeru testno statistiko T primerjamo s konstantno vrednostjo T_{crit} = 1,96 in ne s faktorjema 3 ali 5, kar je v praksi pogosto. Če je za naročnika 5 % tveganje sprejemljivo, to v praksi pomeni, da lahko veliko prej premik neke točke obravnavamo kot statistično značilen. Pri testiranju značilnih premikov je to dejstvo izrednega pomena, saj je cilj odkrivanje značilnih premikov s čim večjo verjetnostjo.

Da bi izračunali najverjetnejšo kritično vrednost pri izbranih stopnjah značilnosti testa, smo testirali različne geometrije mrež na objektih v Sloveniji (glej tabele v prilogi). V preglednici 1 navajamo minimalne in maksimalne kritične vrednosti ter srednjo vrednost T^{mean}_{crit} v testnih mrežah na podlagi simulirane empirične porazdelitvene funkcije za izbrane stopnje značilnosti testa α = 0,10; α = 0,05; α = 0,01 in

α = 0,001. V preglednici 1 navajamo še stopnjo značilnosti α, ki izpolnjuje kriterij mejne vrednosti 3, ki se pri obravnavi značilnih premikov pogosto pojavlja v praksi. Mejne vrednosti 5 ne navajamo, saj je tveganje manjše od 0,001, kar lahko obravnavamo kot odločitev brez tveganja.

Kot vidimo iz preglednice 1, se z zmanjševanjem stopnje značilnosti testa oziroma tveganja za neupravičeno zavrnitev ničelne hipoteze (napaka I. vrste) kritična vrednost ustrezno povečuje. Iz preglednice 1 je razvidno, da je pri primerjavi testne statistike T s kritično vrednostjo T^{mean}_{crit} = 3,59 za 2D-mreže in T^{mean}_{crit} = 3,28 za 3D-mreže tveganje minimalno in znaša 0,1 %. V praksi pogosto navajajo 95 % verjetnost kot sprejemljivo verjetnost pri odločitvi, ali neki premik obravnavamo kot statistično značilen ali ne. V tem primeru testno statistiko T, ki se porazdeljuje po simulirani porazdelitveni funkciji, primerjamo z vrednostjo T^{mean}_{crit} = 2,36 za 2D-mreže in T^{mean}_{crit} = 2,02 za 3D-mreže, in ne s faktorjema 3 ali 5, kar je v praksi pogosto. Če je za naročnika 5 % tveganje sprejemljivo, to v praksi pomeni, da lahko premik neke točke veliko prej obravnavamo kot statistično značilen. Pri testiranju značilnih premikov je to dejstvo izrednega pomena, saj je cilj odkrivanje značilnih premikov s čim večjo verjetnostjo. Glede na posledice nepravilne odločitve se naročnik odloči za sprejemljivo tveganje. V postopku ugotavljanja premikov je cilj statističnega testiranja čim bolj zanesljivo odkrivanje značilnih premikov. Kot vidimo, so nam postopki simulacije v veliko pomoč, saj je pravilno določena porazdelitvena funkcija odločilna pri izračunu natančne kritične vrednosti, na podlagi katere neko točko obravnavamo kos stabilno ali ne.

5 SKLEP

Zanesljivo ocenjevanje opazovanj, neznank in premikov je zelo pomembno, saj imamo v deformacijski analizi opravka z večjim številom opazovanj v več terminskih izmerah. S statističnimi testi ocenjujemo parametre, ugotavljamo skladnost opazovanj in matematičnega modela, odkrivamo grobe pogreške v opazovanjih ter ugotavljamo skladnost domnevno stabilnih točk med terminskimi izmerami in določamo statistično značilne premike nestabilnih točk. Pomembno je, da s postopki statističnega testiranja izločimo možnost, da bi premik neke točke neupravičeno obravnavali kot premik zaradi neskladja med opazovanji in matematičnim modelom ali zaradi neskladja med dvema terminskima izmerama. V tem pogledu je statistično testiranje zelo uporabno orodje, ki nam pomaga pri odločitvah. Ko zagotovimo homogeno natančnost med obravnavanimi terminskimi izmerami, je pomembno, da pri nadaljnji obravnavi premikov uporabimo reprezentativno testno statistiko, s katero bo mogoče premike določiti kar se da zanesljivo. V članku smo podrobneje obravnavali pogosto uporabljeno testno statistiko $T = d/\sigma_d$. V 2D- in 3D-geodetski mreži je premik nelinearna funkcija spremenljivk Δy, Δx in ΔH, zato se ne porazdeljuje po nobeni od znanih porazdelitvenih funkcij.

Uporabnost predlaganega postopka smo testirali na več testnih geodetskih mrežah, ki so se med seboj razlikovale po velikosti in razsežnosti mreže (1D, 2D, 3D), geometriji, številu domnevno stabilnih točk in številu terminskih izmer. V vseh primerih smo simulirali empirično porazdelitveno funkcijo in izračunali kritične vrednosti pri najpogosteje uporabljenih stopnjah značilnosti testa α = 0,10; α = 0,05; α = 0,01 in α = 0,001. Še posebej nas je zanimal v praksi pogosto uporabljen približni kriterij $T > 3$ ali $T > 5$. Ker je empirična porazdelitvena funkcija za vsako točko različna, je različna tudi kritična vrednost. Dodatno so kritične vrednosti ob enakem tveganju različne za 1D-, 2D- ali 3D-geodetsko mrežo.

Ker je kritična vrednost spremenljivka, ni smiselno, da jo obravnavamo kot konstantno vrednost 3 ali

5. V vseh testnih primerih smo ugotovili, da kritična vrednost v vseh dimenzijah mreže doseže vrednost 3 pri izjemno majhnem tveganju, manjšem od 1 %. Stopnjo tveganja 1 % uporabimo pri statističnem testiranju v raziskavah, pri oceni tveganja in pri ugotavljanju premikov kritične infrastrukture (nuklearne elektrarne ipd.). Za večino geotehničnih objektov (mostovi, pregrade idr.) zadošča 95-odstotna zanesljivost ugotavljanja premikov, kar pomeni, da značilen premik lahko odkrijemo že prej. Za naročnika je veliko bolj uporabno, da, glede na ocenjeno tveganje, določimo empirično porazdelitveno funkcijo in s tem tudi natančno kritično vrednost, ki jo primerjamo z izračunano testno statistiko. Pri oceni premikov je konstantno tveganje v vseh dimenzijah geodetske mreže še posebej pomembno, saj tako zagotovimo primerljivost med terminskimi izmerami.

Literatura in viri:

Glej literaturo na strani 397.

Savšek S. (2017). An alternative approach to testing displacements in a geodetic network. Alternativna metoda testiranja premikov v geodetski mreži. Geodetski vestnik, 61 (3), 387-411. DOI: 10.15292/geodetski-vestnik.2017.03.387-411

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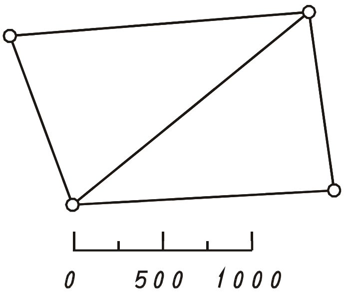
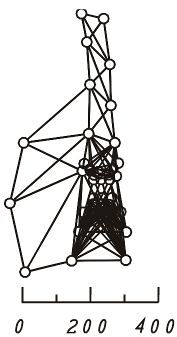
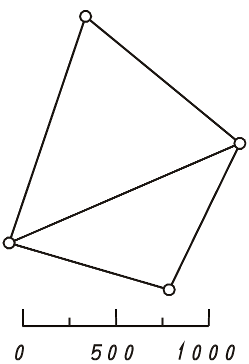
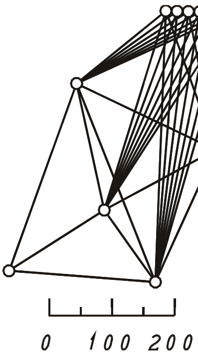
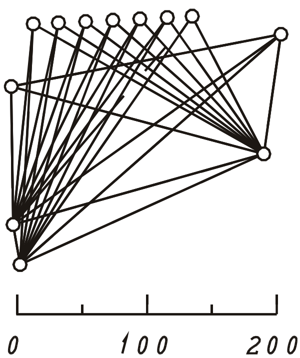
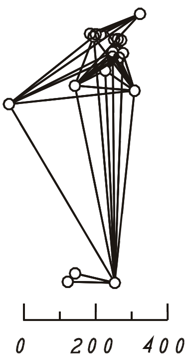
Priloga

Oblika geodetske mreže		Število smeri	Število dolžin	Število zenitnih razdalj	Število neznank	Število točk	Srednja vrednost σ_p (mm)	α	T^{min}_{crit}	T^{max}_{crit}	T^{mean}_{crit}
	2D	10	5		12	4	0,2	0,10	2,01	2,12	2,07
								0,05	2,30	2,42	2,63
								0,01	2,86	3,00	2,93
								0,008	2,92	3,08	3,00
								0,001	3,49	3,68	3,59
								0,00001	4,60	4,95	4,75
	3D	10	5	10	16	4	44,8	0,10	1,64	1,64	1,64
								0,05	1,96	1,96	1,96
								0,01	2,57	2,57	2,57
								0,0034	3,00	3,00	3,00
								0,001	3,23	3,23	3,23
								0,00001	4,06	4,06	4,06
	2D	93	121		99	41	0,3	0,10	1,77	2,14	2,04
								0,05	2,06	2,44	2,33
								0,01	2,65	3,02	2,91
								0,0075	2,75	3,12	3,00
								0,001	3,33	3,69	3,56
								0,00001	4,47	4,97	4,80
	3D	93	121	162	140	41	2,1	0,10	1,70	2,18	1,81
								0,05	2,01	2,46	2,10
								0,0171	2,61	3,01	2,69
								0,01	2,93	3,31	3,00
								0,001	3,27	3,64	3,34
								0,00001	4,06	4,75	4,14
	2D	10	5		12	4	0,5	0,10	2,06	2,14	2,10
								0,05	2,35	2,43	2,39
								0,01	2,93	3,02	2,98
								0,0093	2,95	3,05	3,00
								0,001	3,58	3,70	3,64
								0,00001	4,55	4,85	4,70
	3D	10	5	10	16	4	30,7	0,10	1,64	1,64	1,64
								0,05	1,96	1,96	1,96
								0,01	2,57	2,57	2,57
								0,0027	3,00	3,00	3,00
								0,001	3,23	3,23	3,23
								0,00001	4,06	4,06	4,06

Oblika geodetske mreže		Število smeri	Število dolžin	Število zenitnih razdalj	Število neznank	Število točk	Srednja vrednost σ_p (mm)	α	T_{crit}^{min}	T_{crit}^{max}	T_{crit}^{mean}
	2D	44	34		29	12	0,2	0,10	1,89	2,11	2,00
								0,05	2,18	2,41	2,29
								0,01	2,74	2,99	2,85
								0,0062	2,88	3,15	3,00
								0,001	3,37	3,64	3,50
								0,00001	4,51	4,90	4,67
	3D	44	34	44	41	12	3,1	0,10	1,66	1,67	1,66
								0,05	1,97	1,99	1,98
								0,01	2,58	2,59	2,58
								0,0063	3,00	3,01	3,00
								0,001	3,23	3,25	3,24
								0,00001	4,06	4,07	4,06
	2D	39	29		29	12	0,2	0,10	2,02	2,13	2,10
								0,05	2,30	2,43	2,39
								0,01	2,86	3,01	2,97
								0,0090	2,90	3,04	3,00
								0,001	3,50	3,68	3,62
								0,00001	4,45	4,95	4,82
	3D	39	28	36	41	12	1,5	0,10	1,71	1,94	1,79
								0,05	2,02	2,22	2,09
								0,01	2,62	2,77	2,67
								0,0023	2,94	3,08	3,00
								0,001	3,27	3,41	3,33
								0,00001	4,07	4,21	4,09
	2D	76	45		51	19	0,2	0,10	1,87	2,14	2,08
								0,05	2,16	2,44	2,37
								0,01	2,71	3,03	2,95
								0,0085	2,76	3,08	3,00
								0,001	3,34	3,70	3,61
								0,00001	4,25	4,97	4,80
	3D	76	45	76	70	19	2,4	0,10	1,66	1,76	1,72
								0,05	1,97	2,06	2,02
								0,0188	2,58	2,65	2,62
								0,01	2,96	3,04	3,00
								0,001	3,23	3,31	3,29
								0,00001	4,03	4,10	4,07